# COMPARATIVE STUDY OF CONVENTIONAL SLAB AND FLAT PLATE SYSTEM 

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# COMPARATIVE STUDY OF CONVENTIONAL SLAB AND FLAT PLATE SYSTEM 

## A Thesis

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## DECLARATION

The work performed in this thesis for the achievement of the degree of Bachelor of Science in Civil Engineering is "A study on comparative study of conventional slab and flat plate system". The whole work is carried out by the authors under the strict and friendly supervision of Dr. Enamur Rahim Latifee, Associate Professor, Department of Civil Engineering, Ahsanullah University of Science and Technology, Dhaka, Bangladesh.
Neither this thesis nor any part of it is submitted or is being simultaneously submitted for any degree at any other institutions.

## TO OUR BELOVED PARENTS AND TEACHERS

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#### Abstract

In terms of traditional building design, Slab selection and design becomes the major issue of concern. In most of the cases design and selection of slab depends mainly on economy and of course safety. It is often considered as a choice of selection for less cost and less material while constructing the floor slabs. In this paper, "Comparative Study of Conventional Slab and Flat plate system" comparison between conventional slab system (i.e. Two Way Slab with beams) and Flat plate (i.e. Two Way Slab without beams) is carried out on the basis of economy. Basically, this paper is focused on the comparison in materials between the two slab systems. On the other hand, comparison is made on the total cost (including material cost, laborer cost, form work cost etc.) between them. For the design purpose "Direct Design Method" is used for both the slab systems. The design examples were performed on the same interior and exterior panels with the same long and short dimensions. Before performing the manual calculations, some excel worksheets were developed according to the ACI 318 codes conforming to "Direct Design Method". Then the manual calculations were performed using the same parameters like concrete strength, yield strength, rebar diameter etc. Finally the results are cross-checked and the objectives were compared to one another. For example, in manual calculations minimum thickness of slab is calculated from the thickness table (ACI code 9.5c). However, in excel worksheets thickness was calculated from four different criteria such as effective depth, punching shear mechanism, deflection control mechanism and maximum steel ratio. Different comparison graphs and charts were developed on the basis of determined results. It is seen that up to a certain $\beta$ (long to short span ratio) flat plate is economical over the conventional slab system. However, with further increase in beta $\beta$ the traditional slab system dominates the economy. These comparison results shows a clear overview enough to select the most economical slab system.


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## LIST OF SYMBOLS AND ABBREVIATIONS

$\mathrm{ACI}=$ American Concrete Institute.
ASCE = American Society of Civil Engineers.
BNBC = Bangladesh National Building Code.
$\mathrm{IBC}=$ International Building Code.
UBC = Uniform Building Code.
EN = Euro/British Code.
DDM =Direct Design Method.
EFM = Equivalent Frame Method.
MCM=Moment Coefficient Method
DL = Dead load.
LL = Live load.
I = Length of span.
In = Length of clear span.
$\mathrm{IA}=$ Length of clear span in short direction.
$\mid B=$ Length of clear span in long direction.
As = Area of steel.
$f^{\prime} c=$ Compressive strength of concrete.
fy = Grade/Yield strength of steel.
psi =Pound per square inch
psf = Pound per square foot
ksi $=$ Kilo pound per square inch
ksf = Kio pound per square foot
$h=$ Thickness of slab.
$\mathrm{b}_{\mathrm{w}}=$ beam width
$h_{w}=$ clear height
$\alpha=$ Stiffness ratio of beam to slab.
$\beta=$ Long clear span to short clear span ratio
$E=$ Modulus of elasticity.
I = Moment of inertia.
$\mathrm{P}=$ Perimeter.
$\rho=$ Steel ratio.

Chapter 1 INTRODUCTION

A very common part of any building structure is slab, a structural member that takes the gravity loads in the first place for the entire service period and transmits them to other structural members, like girder, beam, column, etc. When a building has to be designed, engineers analyze and design the slabs at the first place; since, without the slab analysis, the gravity loads cannot be determined to design the other structural members. For this reason, slab design plays a very important role for designing a building.

The most common type of slab is basically supported by beams, transmitting loads to beams. This conventional slab system occupies a great deal of space. To overcome this problem, another type of slab is used, named 'Flat Plate'.

A flat plate is a one- or two-way system usually supported directly on columns or load bearing walls. It is one of the most common forms of construction of floors in buildings. The principal feature of the flat plate floor is a uniform or near-uniform thickness with a flat soffit which requires only simple formwork and is easy to construct.

In Flat Plate the floor allows great flexibility for locating horizontal services above a suspended ceiling or in a bulkhead. The economical span of a flat plate for low to medium loads is usually limited by the need to control long-term deflection and may need to be sensibly pre-cambered (not overdone) or prestressed. This slab system is supported by columns, transmitting loads directly to the columns; omitting conventional beams having large depths from the floor system. Hence this floor system is very popular nowadays. In our project, we have considered these two types of slabs only, i.e. conventional beam-supported slab and flat plate.

Flat plates are widely used for floor construction in multi-story buildings; it is an economical structural system for medium height residential and office buildings. Punching shear is one of the most critical phenomena for flat plate building systems due to the brittle nature of this failure mode. The region of a slab in the vicinity of a column could fail in shear by developing a failure surface in the form of a truncated cone or pyramid. This type of failure, called punching shear failure, is an undesirable mode of failure that occurs without warning and can lead to progressive collapse of large areas of slab or even complete structures. It is one of the topics of intensive research in recent years by various concrete structure researchers. Additionally, the finite element method for the analyses of punching shear failure of reinforced concrete flat plates has been center of attention recently. In this paper, punching shear is checked for design purpose only but no design for punching shear is provided.This chapter summarizes the experimental investigations and analytical approach by different researchers along with provisions of various building codes.

In conventional slab i.e. two way slab system the load transfers in both directions. So bending also occurs in two principal axis. To resist this bending reinforcements are provided in two layers which are perpendicular to each other. This type of slab is surrounded by beams on all sides. Our analysis based on the 'Direct Design Method' is only limited to the slab designs and its cost comparison. None of the beams or its components are brought into considerations.

Analysis of a slab basically deals with the investigation of the internal bending moment reactions in the slab, due to the external loads. These bending moments are used to design a slab. To determine these bending moments, there are different types of technics available. In our project, a technic, named 'Direct Design Method', was used to analyze slabs. In this method, the slab panels are divided into two types of hypothetical parts, i.e. column strips and middle strips. Then the total amount of moment is calculated, and then it is distributed into positive and negative moments at different strips.

No matter what the structure is made of, reinforced concrete ( RC ), steel or composite material, the slabs are always fabricated as RC members. So, in order to design a slab, the parameters to be calculated are thickness of the slab, and position, orientation, spacing of the reinforcing bars. Analyzed bending moments are used to calculate these parameters.

The design procedure contains multiple steps making it a lengthy process. That is why a simplification was necessary to reduce the design time. Hence some excel files were formulated regarding direct design method, which takes some inputs from users and gives the complete design and material estimations as outputs. For the sake of simplification of design and comparison, the reinforcing bars were not assumed to be bent or cranked, but straight.

The purpose of this project was to compare between conventional slab and flat plate, so that both engineers, architects and other non-technical persons can design them and have a rough idea about the cost comparison between those two types of floor systems for different spanlengths and span-ratios.

Chapter 2 LITERATURE REVIEW

### 2.1 Two Way Slab System

A slab system supported by columns or walls, dimensions $c_{1}, c_{2}$ and $I_{n}$ shall be based on an effective support area defined by the intersection of the bottom surface of the slab, or of the drop panel or shear cap if present, with the largest right circular cone, right pyramid, or tapered wedge whose surfaces are located within the column and the capital or bracket and are oriented no greater than 45 degrees to the axis of the column.

### 2.2 Design Procedure

A slab system shall be designed by any procedure satisfying conditions of equilibrium any geometric compatibility if it is shown that at every section is at least equal to the required strength and all serviceability conditions including limits on deflections are met.

Design of a slab system for gravity loads including the slab beams (if any) between supports and supporting columns or walls forming orthogonal frames, by either Direct Design Method or the Equivalent Frame Method.

For gravity load analysis of two-way slab systems, two analysis methods are given in section 13.6 and 13.7 in ACl 318-11. The specific provisions of both design methods are limited in application to orthogonal frames subjected to gravity loads only. Both methods apply to two-way slabs with beams as well as to flat slabs and flat plates. In both methods the distribution of moments to the critical sections of the slab reflects the effects of reduces stiffness of elements due to cracking and support geometry.

### 2.3 Direct Design Method (ACI 318-11 Sec. 13.6)

The Direct Design Method consists of a set of rules for distributing moments to slab and beam sections to satisfy safety requirements and most serviceability requirements simultaneously. Three fundamental steps are involved as follows:

- Determination of total factored static moment.
- Distribution of total factored static moment to negative and positive sections.
- Distribution of the negative and positive factored moments to the column and middle strips and to the beams, if any. The distribution of moments to column and middle strips is also used in the equivalent frame method.


### 2.3.1 Limitations of Direct Design Method (ACI 318-11 Sec. 13.6.1)

- There shall be a minimum of three continuous spans in each direction.
- Panels shall be rectangular, with a ratio of longer to shorter span centre-to-centre of supports within a panel not greater than 2.
- Successive span lengths centre-to-centre of supports in each direction shall not differ by more than one-third the longer span.
- Offset of columns by a maximum of 10 percent of the span (in direction of offset) from either axis between centrelines of successive columns shall be permitted.
- All loads shall be due to gravity only and uniformly distributed over an entire panel. The unfactored live load shall not exceed two times the unfactored dead load.
- For a panel with beams between supports on all sides, Eq.(13-2) in ACI 318-11 shall be satisfied for beams in the two perpendicular directions.

$$
\begin{equation*}
0.20 \leq \frac{\alpha_{f 1} l_{2}^{2}}{\alpha_{f 2} l_{2}^{2}} \leq 5.0 \tag{13-2}
\end{equation*}
$$

Where $I_{2}$ is the clear span between the supports and $\alpha_{f 1}$ and $\alpha_{f 2}$ are calculated in accordance with Eq. (13-3) in $\mathrm{ACl} 318-11$
$\alpha_{f}=\frac{E_{c b} I_{b}}{E_{c s} I_{s}}$

Where $E_{c b}$ and $E_{c s}$ are the modulus of elasticity of beam and slab respectively and $I_{b}$ and $I_{s}$ are the moment of inertia of beam and slab respectively.

- Moment distribution as permitted by Sec. 8.4 in ACI 318-11 shall not be applied for slab system designed by the Direct Design Method.
- Variations from the limitations of 13.6 .1 shall be permitted if demonstrated by analysis that requirements of 13.5 .1 are satisfied.


### 2.3.2 Total Factored Static Moment for A Span (ACI 318-11 Sec. 13.6.2)

Total factored static moment, $\mathrm{M}_{\mathrm{o}}$, for a span shall be determined in a strip bounded laterally by centreline of panel on each side of centreline of supports.

Absolute sum of positive and average negative factored moments in each direction shall not be less than

$$
M_{o}=\frac{\left(q_{u} \ell_{2}\right)\left(\ell_{n}\right)^{2}}{8}
$$

(ACI Equation 13-4)
$l_{n}=$ transverse width of the strip.
$l_{2}=$ clear span between columns

### 2.3.3 Negative \& Positive Factored Moments (ACI 318-11 Sec. 13.6.3)

Negative factored moments shall be located at the face of rectangular supports. Circular or regular polygon shaped supports shall be treated as square supports with the same area.

In an interior span total static moment, $\mathrm{M}_{\mathrm{o}}$ shall be distributed as follows:
Negative factored moment ------------------- 0.65
Positive factored moment ------------------- 0.35
In an end span total factored static moment, Mo shall be distributed as follows:

Table 2.1: Negative \& Positive Factored Moments (ACl 318-11 Sec. 13.6.3)

|  | (1) | (2) | (3) | (4) | (5) |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | Exterior edge unrestrained | Slab with beams between all supports | Slab without beams between interior supports |  | Exterior edge fully restrained |
|  |  |  | Without edge beam | With edge beam |  |
| Interior negative factored moment | 0.75 | 0.70 | 0.70 | 0.70 | 0.65 |
| Positive factored moment | 0.63 | 0.57 | 0.52 | 0.50 | 0.35 |
| Exterior negative factored moment | 0 | 0.16 | 0.26 | 0.30 | 0.65 |

### 2.3.4 Factored Moments in Column Strips (ACI 318-11 Sec. 13.6.4)

Column strip shall be proportioned to resist the following portions in percent of interior negative factored moments: (ACI Code 13.6.4.1)

Table 2.2: Interior Negative Factored Moments in Column Strips (ACI 318-11 Sec. 13.6.4)

| $\ell_{2} / \ell_{1}$ | 0.5 | 1.0 | 2.0 |
| :---: | :---: | :---: | :---: |
| $\left(\alpha_{11} \ell_{2}\right)=0$ | 75 | 75 | 75 |
| $\left(\alpha_{11} \ell_{2} / l_{1}\right) \geq 1.0$ | 90 | 75 | 45 |

Linear interpolation shall be made for the values shown below.
Equations can be used instead of the two-way interpolation

$$
\%_{\text {int col }}^{-}=75+30\left(\frac{\alpha_{f 1} \ell_{2}}{\ell_{1}}\right)\left(1-\frac{\ell_{2}}{\ell_{1}}\right)
$$

Columns strips shall be proportioned to resist the following portions in percent of exterior negative factored moments: (ACI Code 13.6.4.2)

Table 2.3: exterior negative factored moments ( ACI Code 13.6.4.2)

| $\ell_{2} / \ell_{1}$ |  | 0.5 | 1.0 | 2.0 |
| :---: | :---: | ---: | ---: | ---: |
| $\left(\alpha_{\boldsymbol{n}} \ell_{2} / \ell_{1}\right)=0$ | $\beta_{t}=0$ | 100 | 100 | 100 |
|  | $\beta_{t} \geq \mathbf{2 . 5}$ | 75 | 75 | 75 |
| $\left(\alpha_{\boldsymbol{n}} \ell_{2} / \ell_{1}\right) \geq 1.0$ | $\beta_{t}=0$ | 100 | 100 | 100 |
|  | $\beta_{t} \geq \mathbf{2 . 5}$ | 90 | 75 | 45 |

Linear interpolation shall be made between values shown where $\beta_{\mathrm{t}}$ is calculated in Eq. (13$5)$ and $\boldsymbol{C}$ is calculated in Eq. (13-6)

$$
\begin{gather*}
\beta_{t}=\frac{E_{c b} C}{2 E_{c s} I_{S}}  \tag{13-5}\\
C=\sum\left[\left(1-0.63 \frac{x}{y}\right) \frac{x^{3} y}{3}\right\rceil \tag{13-6}
\end{gather*}
$$

The Constant $\boldsymbol{C}$ for $T$ or $L$ sections shall be permitted to be evaluated by dividing the section into separate rectangular parts, as defined in ACI SEC 13.2.4, and summing the values of $\boldsymbol{C}$ for each part

The percentage of exterior negative design moment resisted by the column (\%-ext. col) strip can be found by the following equation

$$
\%_{\mathrm{ext} \mathrm{col}}^{-}=100-10 \beta_{t}+12\left(\frac{\alpha_{f 1} \ell_{2}}{\ell_{1}}\right)\left(1-\frac{\ell_{2}}{\ell_{1}}\right)
$$

## R 13.2.4 ACI 318-11

For monolithic or fully composite construction, the beam include portions of the slab as flanges. Two examples of the rule are provided in the following figure:


Figure 2.1: Composite section

Columns should be proportioned to resist the following portions in percentage of positive factored moments: (ACI Code 13.6.4.4)

| $\ell_{2} / \ell_{1}$ | 0.5 | 1.0 | 2.0 |
| :---: | :---: | :---: | :---: |
| $\left(\alpha_{n} \ell_{2} / \ell_{1}\right)=0$ | 60 | 60 | 60 |
| $\left(\alpha_{11} \ell_{2} / \ell_{1}\right) \geq 1.0$ | 90 | 75 | 45 |

Table 2.4: Positive Factored Moments in Column Strips (ACI 318-11 Sec. 13.6.4.4)

Linear interpolations shall be made for the values shown below.
Finally, for positive design moment in either an interior or exterior span the percentage resisted by the column strip (\%+) is given by the following equation

$$
\%^{+}=60+30\left(\frac{\alpha_{f 1} \ell_{2}}{\ell_{1}}\right)\left(1.5-\frac{\ell_{2}}{\ell_{1}}\right)
$$

For slabs with beams within supports, the slab portion of column strips shall be proportioned to resist that portion of the column strip moments not resisted by beams.

### 2.3.5 Factored Moments in Beam (ACl 318-11 Sec. 13.6.5)

Beams between supports shall be proportioned to resist 85 percent of column strip moments if $\alpha_{f 1} 1_{2} / l_{1}$ is equal to or greater than 1.

For values of $\alpha_{f 1} 1_{2} / l_{1}$ between 1.0 and zero, proportion of column strip moments resisted by beams shall be obtained by linear interpolation between 85 and zero percent.

### 2.3.6 Factored Moments in Middle Strips (ACI 318-11 Sec. 13.6.6)

That proportion of positive and negative moments not resisted by column strips shall be proportionately assigned to its two half middle strips.

Each middle strip shall be proportioned to resist the sum of the moments assigned to its two half middle strips.

A middle strip adjacent to and parallel with a wall supported edge shall be proportioned to resist the twice the moment assigned to the half middle strip corresponding to the first row of interior supports.

### 2.4 Slab Reinforcement (ACI 318-11 Sec. 13.3)

Area of reinforcement in each direction for two-way slab systems shall be determined from moments at critical sections, but shall not be less than required by 7.12.2.1


Figure 2.2: Ductile design Concept

$$
\mathrm{A}_{\mathrm{s}}=\frac{M}{\phi f y\left(d-\frac{a}{2}\right)} \quad ; \mathrm{a}=\frac{A s f y}{.85 f^{\prime} c b}
$$

## ACI Sec. 7.12.2.1

Area of shrinkage and temperature reinforcement shall provide at least the following ratios of reinforcement area to gross concrete area, but not less than 0.0014

- Slabs grade 40 or 50

Deformed bars used

- Slabs grade 60

Deformed bars or welded wires .0018

- Slabs reinforcement with yield

Stress exceeding 60 ksi measured
With a yield strain of 0.35 percent $\frac{0.0018 \times 60000}{f_{y}}$
Spacing of reinforcement at critical sections shall not exceed two times the slab thickness, except for portions of slab area of cellular or ribbed construction.

Positive moment reinforcement perpendicular to a discontinuous edge shall extend to the edge of the slab and have embedment, straight or hooked, at least 6 inch in spandrel beams, columns or walls.

Negative moment reinforcement perpendicular to discontinuous edge shall be bent, hooked or otherwise anchored in spandrel beams, columns or walls and shall be developed at the face of the support.

### 2.4.1 Details of Reinforcement in Slabs without Beams (ACl 318-11 Sec. 13.3.8)

Reinforcements in slabs without beams shall have minimum extensions as prescribed in Figure:

Where adjacent spans are unequal, extensions of negative moment reinforcement beyond the face of support as prescribed in fig shall be based on requirements of the longer span.


Figure 2.3: Minimum extensions for reinforcement in slab without beams
(ACl 318-11 Fig: 13.3.8)

### 2.5 Concrete Floor System

Numerous types of cast-in-place and precast concrete floor systems are available to satisfy virtually any span and loading condition. Reinforced concrete allows a wide range of structural options and provides cost-effective solutions for multitude of situations-from residential buildings with moderate live loads and spans of about 25 ft . to commercial buildings with heavier live loads and spans ranging from 40 ft . to 50 ft . and beyond. Shorter floor-to-floor heights and inherent fire and vibration resistance are only a few of the many advantages that concrete floor system offer, resulting in significant reductions in both structural and non-structural costs.

Since the cost of the floor system can be a major part of the structural cost of a building, selecting the most effective system for a given set of constraints is vital in achieving overall economy. This is especially important for buildings of low-and medium heights and for buildings subjected to relatively low wind or seismic loads, since the cost of lateral load resistance in these cases is minimal. The information provided below will help in selecting an economical cast-in-place concrete floor system for variety of span lengths and superimposed gravity loads.

The main components of cast-in-place concrete floor systems are concrete, reinforcement and formwork. The cost of concrete, including placing and finishing, usually accounts for about $30 \%$ to $35 \%$ of the overall cost of the floor system. Having the greatest influence on the overall cost of the floor system, which is about $45 \%$ to $55 \%$ of the total cost. The reinforcing steel has the lowest influence on the overall cost.

To achieve overall economy, designers should satisfy the following three basic principles of formwork economy:

- Specify readily available standard form sizes.
- Repeat sizes and shapes of the concrete members whenever possible.
- Strive for simple formwork.


### 2.5.1 Flat Plate System

A flat plate floor system is a two-way concrete slab supported directly on columns with reinforcement in two orthogonal directions. This system has the advantages of simple construction and formwork and a flat ceiling, the latter of which reduces ceiling finishing costs, since the architectural finish can be applied directly to the underside of the slab. Even more cost savings associated with the low-story heights made possible by the shallow floor system.

Flat plate systems are economically viable for short to medium spans and for moderate live loads. Up to live loads about 50 psf , the deflection criteria usually govern, and the economical span length range is 15 ft . to 25 ft . for live loads of 100 psf or more, punching shear stresses at the columns and bending moments in the slab control the design. For these cases, the flat plat is economical for spans about between 15 ft . to 20 ft . A flat plate floor with a live load of 100 psf is only about $8 \%$ more expensive than one with a live load of 50 psf, primarily due to the minimum thickness requirements for deflection. Floor panels with an aspect ratio of 2 would be about $30 \%$ more expensive than panels with aspect ratio of 1 ; the thickness of the rectangular panel is governed by the greater span length, resulting in a loss of economy.

Chapter 3

# METHODOLOGY \& 

EXPERIMENTAL
WORK

### 3.1 Floor-Model Selection and Design Approach

In this thesis-project, a floor-system was considered having nine slab-panels (three panels in each direction), and each panel had same spans in same direction; to be used as residential purpose. The plan, showing column-layout, is shown in figure 3.1. All the cross-sectional dimensions of columns were considered 12 " $\times 12^{\prime \prime}$


Figure 3.1: Plan of column-layout to be designed
Those slabs were designed varying spans of a slab-panel, for different long span to short span ratio, $\beta$, (showing in table 3.1).

Table 3.1: Panel sizes of the floor models been designed

| $\boldsymbol{\beta}=\mathbf{1 . 0 0}$ | $\boldsymbol{\beta}=\mathbf{1 . 2 5}$ | $\boldsymbol{\beta}=\mathbf{1 . 5 0}$ | $\boldsymbol{\beta}=\mathbf{1 . 7 5}$ | $\boldsymbol{\beta}=\mathbf{2 . 0 0}$ |
| :---: | :---: | :---: | :---: | :---: |
| $10^{\prime} \times 10^{\prime}$ | $12.5^{\prime} \times 10^{\prime}$ | $15^{\prime} \times 10^{\prime}$ | $17.5^{\prime} \times 10^{\prime}$ | $20^{\prime} \times 10^{\prime}$ |
| $15^{\prime} \times 15^{\prime}$ | $18.75^{\prime} \times 15^{\prime}$ | $22.5^{\prime} \times 15^{\prime}$ | $26.25^{\prime} \times 15^{\prime}$ | $30^{\prime} \times 15^{\prime}$ |
| $20^{\prime} \times 20^{\prime}$ | $25^{\prime} \times 20^{\prime}$ | $30^{\prime} \times 20^{\prime}$ | $35^{\prime} \times 20^{\prime}$ |  |
| $25^{\prime} \times 25^{\prime}$ | $31.25^{\prime} \times 25^{\prime}$ |  |  |  |
| $30^{\prime} \times 30^{\prime}$ |  |  |  |  |
| $35^{\prime} \times 35^{\prime}$ |  |  |  |  |

Excel worksheets were prepared to design only a frame. The frames were divided as per figure

## 3.2. (ACI code 13.2.1 \& 13.2.2)

After design and estimation of the frames, the material amounts were summed, to estimate the total material quantity of the whole floor-system.

Other design data are given below:
$>$ Tensile yield strength of rebar, $\mathrm{f}_{\mathrm{y}}=60,000 \mathrm{psi}$
> Compressive strength of concrete ( 28 days), $\mathrm{f}_{\mathrm{c}}{ }^{\prime}=3000 \mathrm{psi}$
$>$ Live load $=40$ psf (as per BNBC code 2006)
> Load due to floor-finish $=25 \mathrm{psf}$
> Diameter of rebar $=12 \mathrm{~mm}$



Figure 3.2: Division of slab into frames for design

### 3.2 Flat Plate

Step 1: At first, we studied how to design a flat-plate by Direct Design Method, from textbooks and ACl codes.

Step 2: We designed one interior frame and one exterior frame, by hand calculation, to get familiar with the design procedure. Those calculations are shown in article 3.2.1 and 3.2.2.

Step 3: After understanding the design procedure properly, we started to formulate excel spreadsheets, based on what we learnt on previous two steps. Two separate spreadsheets were formulated to design both interior and exterior frames. Those were prepared in way that it would take c/c span lengths of a frame, largest spans along both axes of the floor, column cross-sections, slab-thickness, and different design data as inputs; and would give outputs: slab thickness, bending-moments in both column and middle strips, rebar detailing (showing rebar diameter, spacing and cut-off lengths), width of column and middle strips, and estimated materials (weight of reinforcing steel and volume of concrete).

Step 4: As per table 3.1, inputs were put in the spreadsheets to design and estimate the floors. The first task to work with the spreadsheets was the determination of the slab thickness, which were based on four criteria:

- Minimum thickness to control deflections, established by ACl code: table 9.5(c)
- Minimum effective depth for maximum reinforcement ratios, established by ACI code 10.3.5
- Minimum effective depth required for punching shear capacity of concrete, established by ACI code 11.11.2
- $\quad$ Minimum thickness to satisfy ACl code 9.5.3.2(a)

The worksheets were formulated in such a way that users not only can input the slab thickness, but also see suggested thicknesses satisfying those four conditions (based on the input thickness). So, at first, an assumed thickness had been input in the worksheets for the frame having the largest clear span of the floor, and then observing the other suggestions, input thickness was being changed simultaneously. At the last stage, a thickness had been input that satisfied all the criteria, and not exceeding them. Then that thickness was put in the worksheet for all the frames of a same floor.

Step 5: After designing all the frames and estimation of a floor, estimated material quantities from outputs of those spreadsheets, were summed to estimate total material quantity for the designed floor-system. By the same approach, all the floors were estimated shown in table 3.1.

Step 6: Using all the estimated material quantities for different floors, were used to calculate costs. For cost analysis, the considered types of costs are:

- Material cost

Mild steel rebar was used as reinforcement. The cost-rate of mild steel rebar, found in the market, was BDT 50,000 per metric ton. The cost-rate for Ready Mix Concrete (RMC), found in the market, was BDT 250 per cft. [Shah Cement]

- Form-work cost

The cost-rate assumed was BDT 44.40 per cft.

- Labor cost

The cost-rate assumed was BDT 1.00 per cft.
Those three types of costs were summed and expressed as per square-feet of floor area, which were the total cost for a certain floor.

Step 7: Using the total material quantities and total costs, different column-charts had been developed to interpret cost analysis. Those charts are shown in chapter 5.

### 3.2.1 Design Example

Compute the positive and negative moments in the column and middle strips of the exterior panel of the flat plate between columns B and E in Fig-1. The slab supports a superimposed dead load of 25 psf and a service live load of 50 psf . The beam is 12 inch wide by 16 inch in overall depth and is cast monolithically with the slab


Figure 3.3: Typical Exterior Panel (with edge beam)


Figure 3.4: Edge Beam dimension
TABLE 16.1 Minimum Thickness of Slabs without Interior Beams

|  | Without Drop Panels $^{\dagger}$ |  |  | With Drop Panels $^{\dagger}$ |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Exterior Panels |  | Interior <br> Panels | Exterior Panels |  | Interior <br> Panels |
| Yield strength, <br> $\boldsymbol{f}_{y^{\prime}}$, psi $^{\top}$ | Without edge <br> beams | With edge <br> beams ${ }^{\ddagger}$ |  | Without edge <br> beams | With edge <br> beams |  |
| 40,000 | $\frac{\ell_{n}}{}{ }^{[\xi]}$ | $\frac{\ell_{n}}{33}$ | $\frac{\ell_{n}}{36}$ | $\frac{\ell_{n}}{36}$ | $\frac{\ell_{n}}{40}$ | $\frac{\ell_{n}}{40}$ |
| 60,000 | $\frac{\ell_{n}}{30}$ | $\frac{\ell_{n}}{33}$ | $\frac{\ell_{n}}{33}$ | $\frac{\ell_{n}}{33}$ | $\frac{\ell_{n}}{36}$ | $\frac{\ell_{n}}{36}$ |
| 75,000 | $\frac{\ell_{n}}{28}$ | $\frac{\ell_{n}}{31}$ | $\frac{\ell_{n}}{31}$ | $\frac{\ell_{n}}{31}$ | $\frac{\ell_{n}}{34}$ | $\frac{\ell_{n}}{34}$ |

*For values of reinforcement yield strength between the values given in the table, minimum thickness shall be determined by linear interpolation.
${ }^{\dagger}$ Drop panel is defined in ACl Sections 13.3.7 and 13.2.5.
${ }^{\text {TS }}$ Slabs with beams between columns along exterior edges. The value of $\alpha_{f}$ for the edge beam shall not be less than 0.8 .
${ }^{\S}$ For two-way construction, $\ell_{n}$ is the length of the clear span in the long direction, measured face to face of the supports in slabs without beams and face to face of beams or other supports in other cases.

Table 3.2: (ACI Table 9.5c)

## From table 16.1

Minimum thickness of slab (Exterior panel without drop panel with edge beam)

$$
\mathrm{h}=\frac{l_{n}}{33}=\frac{12 \times\left(21-\frac{14}{2 \times 12}-\frac{16}{2 \times 12}\right)}{33}=7.18 \mathrm{inch} \approx 7.50 \mathrm{in} .
$$

Hence, effective depth, $\mathrm{d}=7.5-1=6.5 \mathrm{in}$.

Factored load $=1.2 \times\left(\frac{7.50}{12} \times 150+25\right)+1.6 \times 50=0.223 \mathrm{ksf}$

## Moments in the long span of the slab:

Statical Moment,

$$
\begin{equation*}
M_{o}=\frac{\left(q_{u} \ell_{2}\right)\left(\ell_{n}\right)^{2}}{8} \tag{ACIEquation13-4}
\end{equation*}
$$

$l_{n}=$ transverse width of the strip.
$l_{2}=$ clear span between columns
$l_{n}=21-\frac{14}{2 \times 12}-\frac{16}{2 \times 12}=19.75 \mathrm{ft}$
$l_{2}=19 \mathrm{ft}$
$M_{0}=\frac{0.223 \times 19 \times 19.75^{2}}{8}=206.58 \mathrm{k}-\mathrm{ft}$

Divide $M_{0}$ into negative and positive moments:
The distribution of the moment to negative and positive regions is as given in Table 16.2 (ACI Sec. 13.6.3.3). In the terminology of Table 16.2 this is a "slab without beams between interior supports with edge beam".

TABLE 16.2 Distribution of Total Span Moment in an End Span (ACI Code 13.6.3.3)

|  | (1) | (2) | (3) | (4) | (5) |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | Slab With between In | t Beams <br> Supports |  |
|  | Exterior Edge <br> Unrestrained | Slab with Beams between All Supports | Without Edge Beam | With Edge Beam | Exterior Edge Fully Restrained |
| Interior negative factored moment | 0.75 | 0.70 | 0.70 | 0.70 | 0.65 |
| Positive factored moment | 0.63 | 0.57 | 0.52 | 0.50 | 0.35 |
| Exterior negative factored moment | 0 | 0.16 | 0.26 | 0.30 | 0.65 |

Table 3.3: (ACI Sec. 13.6.3.3)

From the Table 16.2, the total moment is divided as
Interior negative $\mathrm{M}_{\mathrm{o}}=0.70 \mathrm{M}_{\mathrm{o}}=0.70 \times 206.58=144.61 \mathrm{k}$ - ft
Positive $\mathrm{M}_{\mathrm{o}}=0.50 \mathrm{M}_{\mathrm{o}}=0.50 \times 206.58=103.29 \mathrm{k}-\mathrm{ft}$
Exterior negative $\mathrm{M}_{\mathrm{o}}=0.30 \mathrm{M}_{\mathrm{o}}=0.30 \times 206.58=61.98 \mathrm{k}$ - ft
Division of the moments between column strip and middle strip:
Interior negative moments:
This division is a function of $\alpha_{1} l_{2} / l_{1}$ (taken equal to zero since $\alpha_{1}=0$; because there are no beams parallel tol $l_{1}$ ). From ACI Sec. 13.6.4.1

Interior Column strip negative moment $=0.75 \times 144.61=108.46 \mathrm{k}-\mathrm{ft}$
Interior Middle strip negative moment $=0.25 \times 144.61=36.16 \mathrm{k}$-ft

TABLE 16.3 Percentages of Interior Negative Design
Moments to Be Resisted by Column Strip

| $\frac{\ell_{2}}{\ell_{1}}$ | 0.5 | 1.0 | 2.0 |
| :---: | :---: | :---: | :---: |
| $\frac{\alpha_{f 1} \ell_{2}}{\ell_{1}}=0$ | 75 | 75 | 75 |
| $\frac{\alpha_{f 1} \ell_{2}}{\ell_{1}} \geq 1.0$ | 90 | 75 | 45 |

Table 3.4: (ACI Sec. 13.6.4.1)

Positive Moments:
From ACI Sec. 13.6.4.4, where $\alpha_{1} l_{2} / l_{1}=0$
Column strip positive moment $=0.6 \times 103.29=61.98 \mathrm{k}$-ft
Middle strip positive moment $=0.4 \times 103.29=41.32 \mathrm{k}$ - ft

TABLE 16.5 Percentages of Positive Design Moment to Be Resisted by Column Strip

| $\frac{\ell_{2}}{\ell_{1}}$ | 0.5 | 1.0 | 2.0 |
| :---: | :---: | :---: | :---: |
| $\frac{\alpha_{f 1} \ell_{2}}{\ell_{1}}=0$ | 60 | 60 | 60 |
| $\frac{\alpha_{f 1} \ell_{2}}{\ell_{1}} \geq 1.0$ | 90 | 75 | 45 |

Table 3.5: (ACl Sec. 13.6.4.4)

Exterior negative moment:
From ACl Sec. 13.6.4.2 the exterior negative moment is divided as a function of $\alpha_{1} l_{2} / l_{1}$ (taken equal to zero since $\alpha_{1}=0$ because there are no beams parallel to $l_{1}$ )

And $\beta_{t}$, where

$$
\beta_{t}=\frac{E_{c b} C}{2 E_{c s} I_{s}}
$$

Where $E_{c b}$ and $C$ refer to the attached torsional member shown in figure 2 and $E_{c s}$ and $I_{s}$ refer to the strip of slab being designed (the column strip and two half middle strip shown shaded in Fig 1). To compute C, divide the edge beam into rectangles. The two possibilities shown in figure 2 will be considered.


Figure 3.5: edge beam dimension (for $\beta$ t)

For figure 3, (ACI Eq. 13-7) gives
$\mathrm{C}=\Sigma\left[\left(1-0.63 \frac{x}{y}\right) \frac{x^{3} y}{3}\right]$
Where $x=$ shorter length of the rectangle
$y=$ longer length of the rectangle
For 3a,
$C=\frac{\left.\left(1-0.63 \times \frac{12}{16}\right) \times 12^{3} \times 16\right)}{3}+\frac{\left.\left(1-0.63 \times \frac{7.5}{8.5}\right) \times 7.5^{3} \times 8.5\right)}{3}=5392.30 \mathrm{in}^{4}$
For 3b,
$C=\frac{\left.\left(1-0.63 \times \frac{7.5}{20.5}\right) \times 7.5^{3} \times 20.5\right)}{3}+\frac{\left.\left(1-0.63 \times \frac{8.5}{12}\right) \times 8.5^{3} \times 12\right)}{3}=3578.65 \mathrm{in}^{4}$
The larger of these two values are used.
$\mathrm{C}=5392.30 \mathrm{in}^{4}$
$I_{s}$ is the moment of inertia of the strip of slab being designed. It has $\mathrm{b}=19 \mathrm{ft}$ and $\mathrm{h}=7.5$ inch
$I_{S}=\frac{(19 \times 12) \times 7.5^{3}}{12}=8015.63 \mathrm{in}^{4}$
Since $f_{c}^{\prime}$ is the same in the slab and beam, $E_{c b}=E_{c s}$ and $\beta_{t}=\frac{5392.30}{2 * 8015.63}=0.336$
Interpolating in the table given in $\mathrm{ACl} \mathrm{Sec}$. 13.6.4.2

TABLE 16.4 Percentages of Exterior Negative Design Moment to Be Resisted by Column Strip

| $\frac{\ell_{2}}{\ell_{1}}$ |  | 0.5 | 1.0 | 2.0 |
| :---: | :---: | :---: | :---: | :---: |
| $\frac{\alpha_{f 1} \ell_{2}}{\ell_{1}}=0$ | $\beta_{t}=0$ | 100 | 100 | 100 |
|  | $\beta_{t} \geq 2.5$ | 75 | 75 | 75 |
| $\frac{\alpha_{f 1} \ell_{2}}{\ell_{1}} \geq 1.0$ | $\beta_{t}=0$ | 100 | 100 | 100 |
|  | $\beta_{t} \geq 2.5$ | 90 | 75 | 75 |

Table 3.6: (ACI Sec. 13.6.4.2)

For $\beta_{t}=2.5 ; 75 \%$ to column strip
For $\beta_{t}=0 ; 100 \%$ to column strip

Therefore for $\beta_{t}=0.336 ; 96.64 \%$ to column strip.
Exterior Column strip negative moment $=0.9664 \times 61.98=59.90 \mathrm{k}-\mathrm{ft}$
Exterior Middle strip negative moment $=0.0336 \times 61.98=2.08 \mathrm{k}-\mathrm{ft}$



Figure 3.6: Assignment of $M_{0}$ to positive and negative moment regions in the exterior span

## Shear Check:

(1) Punching shear, $\mathrm{V}_{\mathrm{p}}=[(19 * 21)-(16+6.5) *(14+6.5) / 144] * 0.223=88.27 \mathrm{kip}$

Allowable shear, $\mathrm{V}_{\mathrm{a}}=\phi \mathrm{V}_{\mathrm{c}}=0.75^{*}\left[4 \sqrt{f^{\prime} c} \mathrm{~b}_{0} \mathrm{~d}\right]$
$=0.75 *[4 * \sqrt{3000} *\{(16+6.5+14+6.5) * 2\} * 6.5]$
$=91853 \mathrm{lb}=91.85 \mathrm{kip}>\mathrm{V}_{\mathrm{p}}$ (OK)
(2) Beam shear, $\mathrm{V}_{\mathrm{b}}=\left(\frac{21}{2}-\frac{16}{2 * 12}-\frac{6.5}{12}\right) * 0.223=2.08 \mathrm{kip} / \mathrm{ft}$

Allowable shear, $\mathrm{V}_{\mathrm{a}}=\phi \mathrm{V}_{\mathrm{c}}=0.75 *\left[2 \sqrt{f^{\prime} c} \mathrm{~b}_{\mathrm{w}} \mathrm{d}\right]$

$$
\begin{aligned}
& =0.75 *[2 * \sqrt{3000} * 12 * 6.5]=6408 \mathrm{lb} / \mathrm{ft} \\
& =6.4 \mathrm{kip} / \mathrm{ft}>\mathrm{V}_{\mathrm{b}}(\mathrm{OK})
\end{aligned}
$$

## Calculation of Steel:

(i) Column strip:

Width of column strip $=\frac{l \min }{4}+\frac{l \min }{4}=\frac{18}{4}+\frac{20}{4}=9.5 \mathrm{ft}$
$M_{\text {-ve, interior }}=108.46 / 9.5=11.42 \mathrm{k}-\mathrm{ft} / \mathrm{ft}$
$M_{\text {+ve }}=61.98 / 9.5=6.53 \mathrm{k}-\mathrm{ft} / \mathrm{ft}$
$M_{\text {-ve, exterior }}=59.9 / 9.5=6.31 \mathrm{k}-\mathrm{ft} / \mathrm{ft}$
$\rho_{\max }=0.85 \beta_{1} \frac{f \prime c}{f y} \frac{.003}{.003+.004}=0.85 * 0.85 * 3 / 60 * .003 / .007=0.015$
$\mathrm{d}_{\mathrm{req}}{ }^{2}=\frac{M u}{\Phi f y b \rho\left(1-.59 \rho \frac{f y}{f^{\prime} c}\right)}=\frac{11.42 * 12}{.9 * 60 * 12 * .015 *\left(1-.59 * .015 * \frac{60}{3}\right)}$
So, $d_{\text {req }}=4.14^{\prime \prime}<6.5^{\prime \prime}$ (OK)
$\mathrm{A}_{\mathrm{s}, \min }=.0018 \mathrm{bt}=.0018^{*} 12 * 7.5=0.162 \mathrm{in}^{2} / \mathrm{ft}$

$$
\begin{aligned}
& \text { Now, }+\mathrm{A}_{\mathrm{s}}=\frac{+M}{\phi f y\left(d-\frac{a}{2}\right)} \quad ; \mathrm{a}=\frac{\text { As } f y}{.85 f^{\prime} c b} \\
& =\frac{6.53 * 12}{.9 * 60 *\left(6.5-\frac{4536}{2}\right)} \quad=\frac{0.232 * 60}{.85 * 3 * 12}=0.4536^{\prime \prime} \\
& =0.232 \quad \mathrm{in}^{2} / \mathrm{ft}>\mathrm{A}_{\mathrm{s}, \text { min }}(\mathrm{OK})
\end{aligned}
$$

Using $\phi 10 \mathrm{~mm}$ bar ( $\mathrm{A}_{\mathrm{b}}=0.121 \mathrm{in}^{2}$ ),
Spacing $=\frac{.121}{.232} * 12 \approx 6 \prime<2 \mathrm{~d}$ (OK)
Again, $-\mathrm{A}_{\mathrm{s}, \text { interior }}=\frac{-M}{\phi f y\left(d-\frac{a}{2}\right)} \quad ; \mathrm{a}=\frac{A s f y}{.85 f^{\prime} c b}$

$$
=\frac{11.42 * 12}{.9 * 60 *\left(6.5-\frac{.817}{2}\right)}
$$

$$
=\frac{.417 * 60}{.85 * 3 * 12}=0.817^{\prime \prime}
$$

$$
=0.417 \quad \mathrm{in}^{2} / \mathrm{ft}
$$

Distance between two cranked bars = 6*2 = 12"
Provided steel $=12 * 0.121 / 12=0.121 \mathrm{in}^{2} / \mathrm{ft}$
So, required steel $=0.417-0.121=0.296 \mathrm{in}^{2} / \mathrm{ft}$
Using $\phi 12 \mathrm{~mm}$ bar ( $\mathrm{A}_{\mathrm{b}}=0.175 \mathrm{in}^{2}$ ),
extra negative reinforcement required $=(0.296 / 0.175)^{*}(12 / 12)=1.7 \approx 2$
$2-\$ 12 \mathrm{~mm}$ extra top between ckd. bars at interior end

$$
\text { Again, } \begin{aligned}
-\mathrm{A}_{\mathrm{s}, \text { exterior }} & =\frac{-M}{\phi f y\left(d-\frac{a}{2}\right)} \quad ; \mathrm{a}=\frac{A s f y}{.85 f^{\prime} c b} \\
& =\frac{6.31 * 12}{.9 * 60 *\left(6.5-\frac{438}{2}\right)} \\
& =0.223 \quad \mathrm{in}^{2} / \mathrm{ft}
\end{aligned} \quad=\frac{.223 * 60}{.85 * 3 * 12}=0.438^{\prime \prime}
$$

Distance between two cranked bars $=6 * 2=12^{\prime \prime}$
Provided steel $=0.121 \mathrm{in}^{2} / \mathrm{ft}$
So, required steel $=0.223-0.121=0.102 \mathrm{in}^{2} / \mathrm{ft}$
Using $\phi 10 \mathrm{~mm}$ bar ( $\mathrm{A}_{\mathrm{b}}=0.121 \mathrm{in}^{2}$ ),
extra negative reinforcement required $=(0.102 / 0.121) *(12 / 12)=0.843 \approx 1$
1- $\phi 10 \mathrm{~mm}$ extra top between ckd. bars at exterior end

## (ii) Middle strip:

Width of middle strip $=19-9.5=9.5 \mathrm{ft}$
$\mathrm{M}_{\text {-ve, interior }}=36.16 / 9.5=3.81 \mathrm{k}-\mathrm{ft} / \mathrm{ft}$
$\mathrm{M}_{+\mathrm{ve}}=41.32 / 9.5=4.35 \mathrm{k}-\mathrm{ft} / \mathrm{ft}$
$\mathrm{M}_{\text {-ve, exterior }}=2.08 / 9.5=0.22 \mathrm{k}-\mathrm{ft} / \mathrm{ft}$
$\mathrm{A}_{\mathrm{s}, \min }=0.162 \mathrm{in}^{2} / \mathrm{ft}$

Now, $+\mathrm{A}_{\mathrm{s}}=\frac{+M}{\phi f y\left(d-\frac{a}{2}\right)} \quad ; \mathrm{a}=\frac{A s f y}{.85 f^{\prime} c b}$

$$
\begin{array}{ll}
=\frac{4.35 * 12}{.9 * 60 *\left(6.5-\frac{.298}{2}\right)} \\
=0.153 \quad \mathrm{in}^{2} / \mathrm{ft}<\mathrm{A}_{\mathrm{s}, \text { min }}
\end{array} \quad=\frac{0.153 * 60}{.85 * 3 * 12}=0.298 \prime \prime
$$

So, $A_{s, \min }=0.162 \mathrm{in}^{2} / \mathrm{ft}$ governs.
Using Using $\phi 10 \mathrm{~mm}$ bar ( $\mathrm{A}_{b}=0.121 \mathrm{in}^{2}$ ),
Spacing $=\frac{.121}{.162} * 12 \approx 8.5^{\prime \prime}<2 d$ (OK)
$\phi 10 \mathrm{~mm}$ @ 8.5" c/c alt. ckd. at bottom

$$
\text { Again, } \begin{array}{rlr}
-\mathrm{A}_{\mathrm{s}, \text { interior }} & =\frac{-M}{\phi f y\left(d-\frac{a}{2}\right)} \quad ; \mathrm{a}=\frac{A s f y}{.85 f^{\prime} c b} \\
& =\frac{3.81 * 12}{.9 * 60 *\left(6.5-\frac{.26}{2}\right)} \\
& =0.133 \quad \mathrm{in}^{2} / \mathrm{ft} & =\frac{.133 * 60}{.85 * 3 * 12}=0.26^{\prime \prime}
\end{array}
$$

Distance between two cranked bars = 8.5*2 = 17"
Provided steel $=12^{*} 0.121 / 17=0.085 \mathrm{in}^{2} / \mathrm{ft}$
So, required steel $=0.133-0.085=0.048 \mathrm{in}^{2} / \mathrm{ft}$
Using $\phi 10 \mathrm{~mm}$ bar ( $\mathrm{A}_{b}=0.121 \mathrm{in}^{2}$ ),
extra negative reinforcement required $=(0.048 / 0.121) *(17 / 12)=0.562 \approx 1$
$1-\$ 10 \mathrm{~mm}$ extra top between ckd. bars at interior end

Again, $-\mathrm{A}_{\mathrm{s}, \text { exterior }}=\frac{-M}{\phi f y\left(d-\frac{a}{2}\right)} \quad ; \mathrm{a}=\frac{A s f y}{.85 f^{\prime} c b}$

$$
\begin{array}{ll}
=\frac{.22 * 12}{.9 * 60 *\left(6.5-\frac{.015}{2}\right)} & =\frac{.008 * 60}{.85 * 3 * 12}=0.015^{\prime \prime} \\
=0.008 \quad \mathrm{in}^{2} / \mathrm{ft}
\end{array}
$$

Distance between two cranked bars $=8.5^{*} 2=17 \prime$
Provided steel $=0.085 \mathrm{in}^{2} / \mathrm{ft}>0.008 \mathrm{in}^{2} / \mathrm{ft}$
So, no extra negative reinforcement is required at exterior end.

The reinforcement details are being shown in the following figure.


Fig 3.7: Reinforcement Details

### 3.2.2 Design Example

Compute the positive and negative moments and required amount of steel in the column and middle strips of the interior panel of a flat plate floor in the both short and long direction shown in Fig-1. The slab supports a superimposed dead load of 25 psf and a service live load of 50 psf . $\mathrm{f}^{\prime} \mathrm{c}=3000 \mathrm{psi}$


Figure 3.8: Typical Interior Panel
TABLE 16.1 Minimum Thickness of Slabs without Interior Beams

|  | Without Drop Panels |  |  | With Drop Panels $^{\dagger}$ |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Exterior Panels |  | Interior <br> Panels | Exterior Panels |  | Interior <br> Panels |
| Yield strength, <br> $f_{y}$, psi | Without edge <br> beams | With edge <br> beams ${ }^{\ddagger}$ |  | Without edge <br> beams | With edge <br> beams |  |
| 40,000 | $\frac{\ell_{n}}{33}$ | $\frac{\ell_{n}}{36}$ | $\frac{\ell_{n}}{36}$ | $\frac{\ell_{n}}{36}$ | $\frac{\ell_{n}}{40}$ | $\frac{\ell_{n}}{40}$ |
| 60,000 | $\frac{\ell_{n}}{30}$ | $\frac{\ell_{n}}{33}$ | $\frac{\ell_{n}}{33}$ | $\frac{\ell_{n}}{33}$ | $\frac{\ell_{n}}{36}$ | $\frac{\ell_{n}}{36}$ |
| 75,000 | $\frac{\ell_{n}}{28}$ | $\frac{\ell_{n}}{31}$ | $\frac{\ell_{n}}{31}$ | $\frac{\ell_{n}}{31}$ | $\frac{\ell_{n}}{34}$ | $\frac{\ell_{n}}{34}$ |

*For values of reinforcement yield strength between the values given in the table, minimum thickness shall be determined by linear interpolation.
${ }^{\dagger}$ Drop panel is defined in ACl Sections 13.3.7 and 13.2.5.
$\ddagger$ Slabs with beams between columns along exterior edges. The value of $\alpha_{f}$ for the edge beam shall not be less than 0.8 .
${ }^{\$}$ For two-way construction, $\ell_{n}$ is the length of the clear span in the long direction, measured face to face of the supports in slabs without beams and face to face of beams or other supports in other cases.

From table 16.1
Minimum thickness of slab ((Interior panel without drop panel)

$$
\begin{aligned}
& \mathrm{h}=\frac{l_{n}}{33}=\frac{12 \times\left(14.5-\frac{12}{12}\right)}{33}=4.91 \text { inch } \approx 5 \text { inch } \\
& \mathrm{d}=5^{\prime \prime}-1^{\prime \prime}=4^{\prime \prime}
\end{aligned}
$$

Factored load $=1.2 \times\left(\frac{5}{12} \times 150+25\right)+1.6 \times 50=0.185 \mathrm{ksf}$

## Moments in the short span of the slab:

Static Moment,

$$
\begin{equation*}
M_{o}=\frac{\left(q_{u} \ell_{2}\right)\left(\ell_{n}\right)^{2}}{8} \tag{ACIEquation13-4}
\end{equation*}
$$

$l_{n}=$ transverse width of the strip.
$l_{2}=$ clear span between columns
$l_{n}=13.17-10 / 12=12.33 \mathrm{ft}$
$l_{2}=14.5 \mathrm{ft}$
$M_{0}=\frac{0.185 \times 14.5 \times 12.33^{2}}{8}=50.98 \mathrm{k}-\mathrm{ft} \approx 51 \mathrm{k}-\mathrm{ft}$

Divide $M_{o}$ into negative and positive moments: From code Sec. 13.6.3.2:

$$
\begin{aligned}
& \text { Negative moment }=0.65 \mathrm{M}_{0}=0.65 \times 51=33.15 \mathrm{k}-\mathrm{ft} \\
& \text { Positive moment }=0.35 \mathrm{M}_{0}=0.35 \times 51=17.85 \mathrm{k}-\mathrm{ft}
\end{aligned}
$$

Divide the moments between column strip and middle strip:
Negative moment: From ACI sec. 13.6.4.1 for $\alpha_{1} l_{2} / l_{1}=0$ (taken equal to zero since $\alpha_{1}=$ Obetween the columns in this panel. (Table : 16.3)

Column strip negative moment $=0.75 \times 33.15=24.86 \mathrm{k}$ - ft
Middle strip negative moment $=0.25 \times 33.15=8.29 \mathrm{k}-\mathrm{ft}$

TABLE 16.3 Percentages of Interior Negative Design
Moments to Be Resisted by Column Strip

| $\frac{\ell_{2}}{\ell_{1}}$ | 0.5 | 1.0 | 2.0 |
| :---: | :---: | :---: | :---: |
| $\frac{\alpha_{f 1} \ell_{2}}{\ell_{1}}=0$ | 75 | 75 | 75 |
| $\frac{\alpha_{f 1} \ell_{2}}{\ell_{1}} \geq 1.0$ | 90 | 75 | 45 |

(ACl sec. 13.6.4.1)
Positive Moments: From ACI Sec. 13.6.4.4 , where $\alpha_{1} l_{2} / l_{1}=0$
Column strip positive moment $=0.6 \times 17.85=10.71 \mathrm{k}-\mathrm{ft}$
Middle strip positive moment $=0.4 \times 17.85=7.14 \mathrm{k}$ - ft

TABLE 16.5 Percentages of Positive Design Moment to Be Resisted by Column Strip

| $\frac{\ell_{2}}{\ell_{1}}$ | 0.5 | 1.0 | 2.0 |
| :---: | :---: | :---: | :---: |
| $\frac{\alpha_{f 1} \ell_{2}}{\ell_{1}}=0$ | 60 | 60 | 60 |
| $\frac{\alpha_{f 1} \ell_{2}}{\ell_{1}} \geq 1.0$ | 90 | 75 | 45 |

(ACl sec. 13.6.4.4)


Moment Diagram for Short Span :


Figure 3.9: Assignment of $M_{0}$ to positive and negative moment regions in the interior span

## Moments in the long span of the slab:

$l_{n}=$ transverse width of the strip.
$l_{2}=$ clear span between columns
$l_{n}=14.50-12 / 12=13.50 \mathrm{ft}$
$l_{2}=13.17 \mathrm{ft}$

Compute $\mathrm{M}_{\mathrm{o}}$ :

$$
\begin{equation*}
M_{o}=\frac{\left(q_{u} \ell_{2}\right)\left(\ell_{n}\right)^{2}}{8} \tag{ACIEquation13-4}
\end{equation*}
$$

$\mathrm{M}_{\mathrm{o}}=\frac{0.185 \times 13.17 \times 13.50^{2}}{8}=55.5 \mathrm{k}-\mathrm{ft}$

Divide $M_{o}$ into negative and positive moments: From code Sec. 13.6.3.2:
Negative moment $=0.65 \mathrm{M}_{\mathrm{o}}=0.65 \times 55.5=36.075 \mathrm{k}-\mathrm{ft}$
Positive moment $=0.35 \mathrm{M}_{\mathrm{o}}=0.35 \times 55.5=19.425 \mathrm{k}-\mathrm{ft}$

Divide the moments between column strip and middle strip:
Negative moment: From ACl sec. 13.6.4.1 for $\alpha_{2} l_{2} / l_{1}=0$ ( taken equal to zero since $\alpha_{2}=0$ between the columns in this panel.(Table : 16.3)Divide the moments between column strip and middle strip:

Column strip negative moment $=0.75 \times 36.075=27.06 \mathrm{k}-\mathrm{ft}$
Middle strip negative moment $=0.25 \times 36.85=9.02 \mathrm{k}-\mathrm{ft}$

TABLE 16.3 Percentages of Interior Negative Design
Moments to Be Resisted by Column Strip

| $\frac{\ell_{2}}{\ell_{1}}$ | 0.5 | 1.0 | 2.0 |
| :---: | :---: | :---: | :---: |
| $\frac{\alpha_{f 1} \ell_{2}}{\ell_{1}}=0$ | 75 | 75 | 75 |
| $\frac{\alpha_{f 1} \ell_{2}}{\ell_{1}} \geq 1.0$ | 90 | 75 | 45 |

(ACl sec. 13.6.4.1)

Positive Moments: From ACl Sec. 13.6.4.4, where $\alpha_{2} l_{2} / l_{1}=0$
Column strip positive moment $=0.6 \times 19.425=11.66 \mathrm{k}$-ft
Middle strip positive moment $=0.4 \times 28.86=7.77 \mathrm{k}-\mathrm{ft}$

TABLE 16.5 Percentages of Positive Design Moment to Be Resisted by Column Strip

| $\frac{\ell_{2}}{\ell_{1}}$ | 0.5 | 1.0 | 2.0 |
| :---: | :---: | :---: | :---: |
| $\frac{\alpha_{f 1} \ell_{2}}{\ell_{1}}=0$ | 60 | 60 | 60 |
| $\frac{\alpha_{f 1} \ell_{2}}{\ell_{1}} \geq 1.0$ | 90 | 75 | 45 |

(ACl sec. 13.6.4.4)


Moment Diagram for Long Span:


Figure 3.10: Assignment of $M_{0}$ to positive and negative moment regions in the interior span

## Shear Check:

(1) Punching Shear $\mathrm{Vp}=\left[(14.5 \times 13.167)-\frac{(12+4) x(10+4)}{144}\right] \times 0.185$

$$
=35.03 \mathrm{kip}
$$

Allowable shear $\mathrm{Va}=\Phi \mathrm{Vc}=0.75 \times 4 \sqrt{f^{\prime} c} \mathrm{~b}_{0} \mathrm{~d}=0.75 \times 4 \times \sqrt{3000} \times\{4 \times(12+4)\} \times 4$

$$
=\frac{42065.09}{1000}=42.1 \mathrm{kip}>\mathrm{Vp}=35.03 \mathrm{kip}(\mathrm{OK})
$$

(2) Beam shear $\mathrm{Vb}=0.185 \times\left(\frac{14.5}{2}-\frac{12}{2 \times 12}-\frac{4}{12}\right)=0.185 \times 6.42=1.187 \mathrm{kip}$ Allowable shear $\mathrm{Va}=\Phi \mathrm{Vc}=0.75 \times 2 \sqrt{f^{\prime} c} \mathrm{~b}_{\mathrm{w}} \mathrm{d}=0.75 \times 2 \times \sqrt{3000} \times 12 \times 4$

$$
\text { =3943.60/1000 = } 3.94 \text { kip > Vb =1.187 kip (OK) }
$$

## Calculation of Steel

In short span:
(i) Column Strip

Width of Column strip $=\frac{l \min }{4}+\frac{l_{\text {min }}}{4}=\frac{13.167}{4}+\frac{13.167}{4}=6.58 \mathrm{ft}$

1. Negative moment (Column strip) $=0.75 \times 33.15=24.86 \mathrm{k}-\mathrm{ft} / 6.58 \mathrm{ft}=3.78 \mathrm{k}-\mathrm{ft} / \mathrm{ft}=\mathrm{M}_{\mathrm{u}}$
2. Positive moment (Column strip) $=0.6 \times 17.85=10.71 \mathrm{k}-\mathrm{ft} / 6.58 \mathrm{ft}=1.63 \mathrm{k}-\mathrm{ft} / \mathrm{ft}$
$\rho_{\text {max }}=0.85 \times \beta_{1} \times \frac{f f c}{f y} \times \frac{.003}{.003+.004}=0.85 \times 0.85 \times 3 / 60 \times .003 / .007=0.015$
$d^{2}$ req $=\frac{M u}{\phi \text { fy } b \rho \max \left(1-\frac{59 \mathrm{fy} \rho \max }{f^{\prime} \mathrm{c}}\right)}=\frac{3.78 \times 12}{.9 \times 60 \times 12 x .015\left(1-\frac{.59 \times 60 \times .015}{3}\right)}=5.67^{\prime \prime}$
So, $d=\sqrt{5.67}=2.38<d_{\text {prov }}=4^{\prime \prime}$ (OK)
As $\min =.0018 \mathrm{bt}=.0018 \times 12 \times 5=.108 \mathrm{in}^{2}=69.68 \mathrm{~mm}^{2}$
So, $+\mathrm{As}=\frac{+M}{\Phi f y\left(d-\frac{a}{2}\right)} \quad ; \mathrm{a}=\frac{A s f y}{.85 f^{\prime} c b}$

$$
=\frac{1.63 \times 12}{.9 \times 60 \times\left(4-\frac{181}{2}\right)}=0.093 \mathrm{in}^{2} \quad=\frac{.093 \times 60}{.85 \times 3 \times 12}=.181 \text { " }
$$

$$
=60 \mathrm{~mm}^{2}<\mathrm{As}_{\text {min }}, \text { So,As } \mathrm{min} \text { govern }\left(69.68 \mathrm{~mm}^{2}\right) . \text { Using } \Phi 10 \mathrm{~mm}, \text { Area }=78.5 \mathrm{~mm}^{2},
$$

spacings $=\frac{78.5 \times 12}{69.68}=13.5^{\prime \prime}=10^{\prime \prime}$ not more than 2 d

Use $\Phi 10 \mathrm{~mm}$ @ 10 " $\mathrm{c} / \mathrm{c}$ alt ckd. at bottom

Again. -As $=\frac{-M}{\Phi f y\left(d-\frac{a}{2}\right)} \quad ; \mathrm{a}=\frac{A s f y}{.85 f^{\prime} c b}$

$$
\begin{aligned}
& =\frac{3.78 \times 12}{.9 \times 60 \times\left(4-\frac{.435}{2}\right)}=0.22 \mathrm{in}^{2} \quad=\frac{.22 \times 60}{.85 \times 3 \times 12}=.435^{\prime \prime} \\
& =143.23 \mathrm{~mm}^{2}>\text { Asmin }
\end{aligned}
$$

So,using $\Phi 12 \mathrm{~mm}$, distance between ckd bars 10x2=20"
Provided Steel $=\frac{12 \times 113}{24}=56.5 \mathrm{~mm}^{2}$; so required steel $=143.23-56.5=86.73 \mathrm{~mm}^{2}$
Using $1 \Phi 12 \mathrm{~mm}$ as extra top, area $=1 \times 113=113 \mathrm{~mm}^{2}>86.73 \mathrm{~mm}^{2}$

Provide $1 \Phi 12 \mathrm{~mm}$ extra top between ckd. bars
(ii) Middle Strip

Width of middle strip $=14.5-6.58=7.92 \mathrm{ft}$

1. Negative moment (Middle strip) $=0.25 \times 33.15=8.29 \mathrm{k}-\mathrm{f} / 7.92 \mathrm{ft}=1.05 \mathrm{k}-\mathrm{ft} / \mathrm{ft}=\mathrm{M}_{\mathrm{u}}$
2. Positive moment $($ Middle strip $)==0.4 \times 17.85=7.14 \mathrm{k}-\mathrm{ft} / 7.92 \mathrm{ft}=0.90 \mathrm{k}-\mathrm{ft} / \mathrm{ft}$
$\rho_{\max }=0.85 \times \beta_{1} \times \frac{f \prime c}{f y} \times \frac{.003}{.003+.004}=0.85 \times 0.85 \times 3 / 60 \times .003 / .007=0.015$
$d^{2}$ req $=\frac{M u}{\Phi \text { fy } b \rho \max \left(1-\frac{.59 \text { fy } \rho \max }{f^{\prime} \mathrm{c}}\right)}=\frac{1.05 \times 12}{.9 \times 60 \times 12 \times .015\left(1-\frac{.59 \times 60 \times .015}{3}\right)}=1.58^{\prime \prime}$
So, $d=\sqrt{1.58}=1.26<d_{\text {prov }}=4^{\prime \prime}(O K)$
As $\min =.0018 \mathrm{bt}=.0018 \times 12 \times 5=.108 \mathrm{in}^{2}=69.68 \mathrm{~mm}^{2}$

$$
\begin{aligned}
\text { So, }+\mathrm{As} & =\frac{+M}{\Phi f y\left(d-\frac{a}{2}\right)} \quad ; \mathrm{a}=\frac{A s f y}{.85 f^{\prime} c b} \\
& =\frac{0.90 \times 12}{.9 \times 60 \times\left(4-\frac{0.1}{2}\right)}=0.051 \mathrm{in}^{2} \quad=\frac{.051 \times 60}{.85 \times 3 \times 12}=0.1^{\prime \prime} \\
& =32.9 \mathrm{~mm}^{2}<\text { Asmin }, \text { So,As min govern }\left(69.68 \mathrm{~mm}^{2}\right) . \text { So,using } \Phi 10 \mathrm{~mm}, \text { Area }=78.5 \mathrm{~mm}^{2}
\end{aligned}
$$

,

Use $\Phi 10 \mathrm{~mm}$ @ 10"c/c alt ckd. at bottom

Again. -As $=\frac{-M}{\Phi f y\left(d-\frac{a}{2}\right)} \quad ; \mathrm{a}=\frac{A s f y}{.85 f^{\prime} c b}$

$$
=\frac{1.05 \times 12}{.9 \times 60 \times\left(4-\frac{0.12}{2}\right)}=0.06 \mathrm{in}^{2} \quad=\frac{.06 \times 60}{.85 \times 3 \times 12}=0.12^{\prime \prime}
$$

$=38.71 \mathrm{~mm}^{2}<\mathrm{As}_{\text {min }}$, So,As $\min$ govern $\left(69.68 \mathrm{~mm}^{2}\right) . U \operatorname{sing} \Phi 10 \mathrm{~mm}$, Area $=78.5 \mathrm{~mm}^{2}$ spacings $=\frac{78.5 \times 12}{69.68}=13.5^{\prime \prime}=10 "$ [not more than $\left.2 d\right]$

$$
\text { Use } \Phi 10 \mathrm{~mm} @ 10^{\prime \prime} \mathrm{c} / \mathrm{c} \text { at top }
$$

In Long Span:
(i) Column Strip

Width of Column strip $=\frac{l \min }{4}+\frac{l \min }{4}=\frac{13.167}{4}+\frac{13.167}{4}=6.58 \mathrm{ft}$

1. Negative moment (Column strip) $=0.75 \times 36.075=27.06 \mathrm{k}-\mathrm{ft} / 6.58 \mathrm{ft}=4.12 \mathrm{k}-\mathrm{ft} / \mathrm{ft}=\mathrm{M}_{\mathrm{u}}$
2.Positive moment (Column strip $)=0.6 \times 19.425=11.66 \mathrm{k}-\mathrm{ft} / 6.58 \mathrm{ft}=1.77 \mathrm{k}-\mathrm{ft} / \mathrm{ft}$
$\rho_{\max }=0.85 \times \beta_{1} \times \frac{f \prime c}{f y} \times \frac{.003}{.003+.004}=0.85 \times 0.85 \times 3 / 60 \times .003 / .007=0.015$
$d^{2}$ req $=\frac{M u}{\Phi \text { fy } b \rho \max \left(1-\frac{.59 \text { fy } \rho \max }{f^{\prime} \mathrm{c}}\right)}=\frac{4.12 \times 12}{.9 \times 60 \times 12 \times .015\left(1-\frac{.59 \times 60 \times .015}{3}\right)}=6.18^{\prime \prime}$
So, $d=\sqrt{6.18}=2.49<d_{\text {prov }}=4^{\prime \prime}$ (OK)
As $\min =.0018 \mathrm{bt}=.0018 \times 12 \times 5=.108 \mathrm{in}^{2}=69.68 \mathrm{~mm}^{2}$

So, + As $=\frac{+M}{\Phi f y\left(d-\frac{a}{2}\right)} \quad ; \mathrm{a}=\frac{A s f y}{.85 f^{\prime} c b}$

$$
=\frac{1.77 \times 12}{.9 \times 60 \times\left(4-\frac{.198}{2}\right)}=0.1 \mathrm{in}^{2} \quad=\frac{.1 \times 60}{.85 \times 3 \times 12}=.198 \text { " }
$$

$=64.52 \mathrm{~mm}^{2}<A s_{\min }$, So,As $\min \operatorname{govern}\left(69.68 \mathrm{~mm}^{2}\right) . U \operatorname{sing} \Phi 10 \mathrm{~mm}, A r e a=78.5 \mathrm{~mm}^{2}$,
spacings $=\frac{78.5 \times 12}{69.68}=13.5^{\prime \prime}=10^{\prime \prime}[$ not more than $2 d]$

Use $\Phi 10 \mathrm{~mm}$ @ 10"c/c alt ckd. at bottom

Again. -As $=\frac{-M}{\Phi \text { fy }\left(d-\frac{a}{2}\right)} \quad ; \mathrm{a}=\frac{A s f y}{.85 f^{\prime} c b}$

$$
\begin{aligned}
& =\frac{4.12 \times 12}{.9 \times 60 \times\left(4-\frac{.48}{2}\right)}=0.24 \mathrm{in}^{2} \quad=\frac{.24 \times 60}{.85 \times 3 \times 12}=.48^{\prime \prime} \\
& =154.84 \mathrm{~mm}^{2}>\mathrm{As}_{\min },
\end{aligned}
$$

So,using $\Phi 12 \mathrm{~mm}$, distance between ckd bars $10 \times 2=20$ "
Provided Steel $=\frac{12 \times 113}{24}=56.5 \mathrm{~mm}^{2} ;$ So. required steel $=154.84-56.5=98.34 \mathrm{~mm}^{2}$
Using $1 \Phi 12 \mathrm{~mm}$ as extra top ,area $=1 \times 113=113 \mathrm{~mm}^{2}>98.34 \mathrm{~mm}^{2}$

Provide $1 \Phi 12 \mathrm{~mm}$ extra top between ckd. bars
(ii) Middle Strip

Width of middle strip $=13.167-6.58=6.587 \mathrm{ft}$

1. Negative moment (Middle strip) $=0.25 \times 36.85=9.02 \mathrm{k}-\mathrm{ft} / 6.587 \mathrm{ft}=1.37 \mathrm{k}-\mathrm{ft} / \mathrm{ft}=\mathrm{M}_{\mathrm{u}}$
2.Positive moment (Middle strip) $==0.4 \times 28.86=7.77 \mathrm{k}-\mathrm{ft} / 6.587 \mathrm{ft}=1.18 \mathrm{k}-\mathrm{ft} / \mathrm{ft}$
$\rho_{\max }=0.85 \times \beta_{1} \times \frac{f \prime c}{f y} \times \frac{.003}{.003+.004}=0.85 \times 0.85 \times 3 / 60 \times .003 / .007=0.015$
$d^{2}$ req $=\frac{M u}{\Phi f y b \rho \max \left(1-\frac{.59 \mathrm{fy} \rho \mathrm{max}}{\mathrm{f}^{\prime} \mathrm{c}}\right)}=\frac{1.37 \times 12}{.9 \times 60 \times 12 \times .015\left(1-\frac{.59 \times 60 \times .015}{3}\right)}=2.1^{\prime \prime}$
So, $d=\sqrt{2.1}=1.45<d_{\text {prov }}=4 \prime$ (OK)
As $\min =.0018 \mathrm{bt}=.0018 \times 12 \times 5=.108 \mathrm{in}^{2}=69.68 \mathrm{~mm}^{2}$

$$
\begin{aligned}
\text { So, }+ \text { As } & =\frac{+M}{\Phi f y\left(d-\frac{a}{2}\right)} \quad ; a=\frac{A s f y}{.85 f^{\prime} c b} \\
& =\frac{1.18 \times 12}{.9 \times 60 \times\left(4-\frac{0.13}{2}\right)}=0.07 \mathrm{in}^{2} \quad=\frac{.07 \times 60}{.85 \times 3 \times 12}=0.13 \text { " } \\
& =45.16 \mathrm{~mm}^{2}<A s_{m i n}, \text { So,As min govern }\left(69.68 \mathrm{~mm}^{2}\right) . U \operatorname{sing} \Phi 10 \mathrm{~mm}, A r e a=78.5 \mathrm{~mm}^{2}
\end{aligned}
$$

spacings $=\frac{78.5 \times 12}{69.68}=13.5^{\prime \prime}=10^{\prime \prime}$ [not more than 2 d$] \quad$ Use $\Phi 10 \mathrm{~mm}$ @ 10 " $\mathrm{c} / \mathrm{c}$ alt ckd. at bottom

Again. -As $=\frac{-M}{\Phi f y\left(d-\frac{a}{2}\right)} \quad ; \mathrm{a}=\frac{A s f y}{.85 f^{\prime} c b}$

$$
=\frac{1.37 \times 12}{.9 \times 60 \times\left(4-\frac{0.152}{2}\right)}=0.08 \mathrm{in}^{2} \quad=\frac{.08 \times 60}{.85 \times 3 \times 12}=0.152^{\prime \prime}
$$

$=51.61 \mathrm{~mm}^{2}<\mathrm{As}_{\min }$, So,As min govern $\left(69.68 \mathrm{~mm}^{2}\right)$,Using $\Phi 10 \mathrm{~mm}$, Area $=78.5 \mathrm{~mm}^{2}$, spacings $=\frac{78.5 \times 12}{69.68}=13.5^{\prime \prime}=10^{\prime \prime}$ [not more than 2 d ]

$$
\text { Use } \Phi 10 \mathrm{~mm} @ 10^{\prime \prime} \mathrm{c} / \mathrm{c} \text { at top }
$$

### 3.2.3 Design Example

Compute the positive and negative moments in the column and middle strips of the exterior panel (without edge beam) of the flat plate between columns in Fig-1. The slab supports a superimposed dead load of 25 psf and a service live load of 50 psf.


Fig 3.11: Typical Exterior Panel (without edge beam)

## ACI Table 9.5c

TABLE 16.1 Minimum Thickness of Slabs without Interior Beams

|  | Without Drop Panels ${ }^{\dagger}$ |  |  | With Drop Panels ${ }^{\dagger}$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Exterior Panels |  | Interior Panels | Exterior Panels |  | Interior Panels |
| Yield strength, $f_{y}, \mathrm{psi}^{*}$ | Without edge beams | With edge beams ${ }^{\ddagger}$ |  | Without edge beams | With edge beams ${ }^{\ddagger}$ |  |
| 40,000 | $\frac{\ell_{n}}{}{ }^{\text {[§] }]}$ | $\frac{\ell_{n}}{36}$ | $\frac{\ell_{n}}{36}$ | $\frac{\ell_{n}}{36}$ | $\frac{\ell_{n}}{40}$ | $\frac{\ell_{n}}{40}$ |
| 60,000 | $\frac{\ell_{n}}{30}$ | $\frac{\ell_{n}}{33}$ | $\frac{\ell_{n}}{33}$ | $\frac{\ell_{n}}{33}$ | $\frac{\ell_{n}}{36}$ | $\frac{\ell_{n}}{36}$ |
| 75,000 | $\frac{\ell_{n}}{28}$ | $\frac{\ell_{n}}{31}$ | $\frac{\ell_{n}}{31}$ | $\frac{\ell_{n}}{31}$ | $\frac{\ell_{n}}{34}$ | $\frac{\ell_{n}}{34}$ |

*For values of reinforcement yield strength between the values given in the table, minimum thickness shall be determined by linear interpolation.
${ }^{\dagger}$ Drop panel is defined in ACl Sections 13.3.7 and 13.2.5.
${ }^{\ddagger}$ Slabs with beams between columns along exterior edges. The value of $\alpha_{f}$ for the edge beam shall not be less than 0.8 .
${ }^{\$}$ For two-way construction, $\ell_{n}$ is the length of the clear span in the long direction, measured face to face of the supports in slabs without beams and face to face of beams or other supports in other cases.

From table 16.1[ACI Table 9.5c]
Minimum thickness of slab (Exterior panel without drop panel without edge beam)

$$
\mathrm{h}=\frac{l_{n}}{30}=\frac{12 \times\left(21-\frac{14}{2 \times 12}-\frac{16}{2 \times 12}\right)}{30}=7.9 \text { inch } \approx 8 \text { inch So, } \mathrm{d}=8-1=7 \text { inch }
$$

Factored load $=1.2 \times\left(\frac{8}{12} \times 150+25\right)+1.6 \times 50=0.230 \mathrm{ksf}$

## Moments in the long span of the slab:

Static Moment ,

$$
\begin{equation*}
M_{o}=\frac{\left(q_{u} \ell_{2}\right)\left(\ell_{n}\right)^{2}}{8} \tag{ACIEquation13-4}
\end{equation*}
$$

$l_{n}=$ transverse width of the strip.
$l_{2}=$ clear span between columns
$l_{n}=21-\frac{14}{2 \times 12}-\frac{16}{2 \times 12}=19.75 \mathrm{ft}$
$l_{2}=19 \mathrm{ft}$
$M_{0}=\frac{0.230 \times 19 \times 19.75^{2}}{8}=213.1 \mathrm{k}-\mathrm{ft}$

Divide $M_{0}$ into negative and positive moments:
The distribution of the moment to negative and positive regions is as given in Table 16.2 (ACI Sec. 13.6.3.3).In the terminology of Table 16.2 this is a "slab without beams between interior supports "without edge beam".

TABLE 16.2 Distribution of Total Span Moment in an End Span (ACI Code 13.6.3.3)

|  | (1) | (2) | (3) | (4) | (5) |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | Slab Without Beams <br> between Interior Supports |  |  |  |
|  |  |  | Without <br> Exterior Edge <br> Unrestrained | Slab with Beams <br> Edge Beam | With Edge <br> Beam | Exterior Edge <br> Fully Restrained |
| Interior negative <br> factored moment | 0.75 | 0.70 | 0.70 | 0.70 | 0.65 |  |
| Positive factored <br> moment | 0.63 | 0.57 | 0.52 | 0.50 | 0.35 |  |
| Exterior negative <br> factored moment | 0 | 0.16 | 0.26 | 0.30 | 0.65 |  |

Table 3.7: Distribution of total moment (without edge beam)

From the Table 13-2 , the total moment is divided as
Interior negative $\mathrm{M}_{\mathrm{o}}=0.70 \mathrm{M}_{\mathrm{o}}=0.70 \times 213.1=149.17 \mathrm{k}-\mathrm{ft}$
Positive $\mathrm{M}_{\mathrm{o}}=0.52 \mathrm{M}_{\mathrm{o}}=0.52 \times 213.1=110.81 \mathrm{k}-\mathrm{ft}$
Exterior negative $\mathrm{M}_{\mathrm{o}}=0.26 \mathrm{M}_{\mathrm{o}}=0.26 \times 213.1=55.41 \mathrm{k}$ - ft

Divide the moments between column strip and middle strip:

## Interior negative moments:

This division is a function of $\alpha_{1} l_{2} / l_{1}$ ( taken equal to zero since $\alpha_{1}=0$ because there are no beams parallel to $l_{1}$ ). From ACI Sec. 13.6.4.1

Interior Column strip negative moment $=0.75 \times 149.17=111.88 \mathrm{k}-\mathrm{ft}$
Interior Middle strip negative moment $=0.25 \times 149.17=37.3 \mathrm{k}$-ft

TABLE 16.3 Percentages of Interior Negative Design
Moments to Be Resisted by Column Strip

| $\frac{\ell_{2}}{\ell_{1}}$ | 0.5 | 1.0 | 2.0 |
| :---: | :---: | :---: | :---: |
| $\frac{\alpha_{f 1} \ell_{2}}{\ell_{1}}=0$ | 75 | 75 | 75 |
| $\frac{\alpha_{f 1} \ell_{2}}{\ell_{1}} \geq 1.0$ | 90 | 75 | 45 |

(ACl Sec. 13.6.4.1)

Positive Moments: From ACI Sec. 13.6.4.4, where $\alpha_{1} l_{2} / l_{1}=0$
Column strip positive moment $=0.6 \times 110.81=66.5 \mathrm{k}-\mathrm{ft}$
Middle strip positive moment $=0.4 \times 110.81=44.32 \mathrm{k}-\mathrm{ft}$

TABLE 16.5 Percentages of Positive Design Moment to Be Resisted by Column Strip

| $\frac{\ell_{2}}{\ell_{1}}$ | 0.5 | 1.0 | 2.0 |
| :---: | :---: | :---: | :---: |
| $\frac{\alpha_{f 1} \ell_{2}}{\ell_{1}}=0$ | 60 | 60 | 60 |
| $\frac{\alpha_{f 1} \ell_{2}}{\ell_{1}} \geq 1.0$ | 90 | 75 | 45 |

(ACl Sec. 13.6.4.4)

## Exterior negative moment:

From ACl Sec. 13.6.4.2 the exterior negative moment is divided as a function of $\alpha_{1} l_{2} / l_{1}$ ( taken equal to zero since $\alpha_{1}=0$ because there are no beams parallel to $l_{1}$ )

And $\beta_{t}$, where

$$
\beta_{t}=\frac{E_{c b} C}{2 E_{c s} I_{s}}
$$

TABLE 16.4 Percentages of Exterior Negative Design Moment to Be
Resisted by Column Strip

| $\frac{\ell_{2}}{\ell_{1}}$ |  | 0.5 | 1.0 | 2.0 |
| :---: | :---: | :---: | :---: | :---: |
| $\frac{\alpha_{f 1} \ell_{2}}{\ell_{1}}=0$ | $\beta_{t}=0$ | 100 | 100 | 100 |
|  | $\beta_{t} \geq 2.5$ | 75 | 75 | 75 |
| $\frac{\alpha_{f 1} \ell_{2}}{\ell_{1}} \geq 1.0$ | $\beta_{t}=0$ | 100 | 100 | 100 |
|  | $\beta_{t} \geq 2.5$ | 90 | 75 | 75 |

(ACl Sec. 13.6.4.2)

As there is No Edge beam $\beta_{t}=0$; so column strip takes $100 \%$ of the exterior negative moment.

Exterior Column strip negative moment $=1.0 \times 55.41=55.41 \mathrm{k}-\mathrm{ft}$



Figure 3.12: Assignment of $M_{0}$ to positive and negative moment regions in the exterior span

## Shear Check:

(3) Punching shear, $\mathrm{V}_{\mathrm{p}}=\left[(19 * 21)-(16+7)^{*}(14+7) / 144\right]^{*} 0.230=91 \mathrm{kip}$

Allowable shear, $\mathrm{V}_{\mathrm{a}}=\phi \mathrm{V}_{\mathrm{c}}=0.75^{*}\left[4 \sqrt{f^{\prime} c} \mathrm{~b}_{0} \mathrm{~d}\right]$
$=0.75 *[4 * \sqrt{3000} *\{(16+7+14+7) * 2\} * 7]$
$=101219.13 \mathrm{lb}=101.22 \mathrm{kip}>\mathrm{V}_{\mathrm{p}}(\mathrm{OK})$
(4) Beam shear, $\mathrm{V}_{\mathrm{b}}=\left(\frac{21}{2}-\frac{16}{2 * 12}-\frac{7}{12}\right) * 0.230=2.13 \mathrm{kip} / \mathrm{ft}$

Allowable shear, $\mathrm{V}_{\mathrm{a}}=\phi \mathrm{V}_{\mathrm{c}}=0.75 *\left[2 \sqrt{f^{\prime} c} \mathrm{~b}_{\mathrm{w}} \mathrm{d}\right]$

$$
\begin{aligned}
& =0.75 *[2 * \sqrt{3000} * 12 * 7]=6901.3 \mathrm{lb} / \mathrm{ft} \\
& =6.9 \mathrm{kip} / \mathrm{ft}>\mathrm{V}_{\mathrm{b}}(\mathrm{OK})
\end{aligned}
$$

## Calculation of Steel:

(iii) Column strip:

Width of column strip $=\frac{l \min }{4}+\frac{l \min }{4}=\frac{18}{4}+\frac{20}{4}=9.5 \mathrm{ft}$
$M_{\text {-ve, interior }}=111.88 / 9.5=11.78 \mathrm{k}$ - $\mathrm{ft} / \mathrm{ft}$
$M_{+v e}=66.5 / 9.5=7.0 \mathrm{k}-\mathrm{ft} / \mathrm{ft}$
$\mathrm{M}_{\text {-ve, exterior }}=55.41 / 9.5=5.83 \mathrm{k}$ - $\mathrm{ft} / \mathrm{ft}$
$\rho_{\max }=0.85 \beta_{1} \frac{f \prime c}{f y} \frac{.003}{.003+.004}=0.85 * 0.85 * 3 / 60 * .003 / .007=0.015$
$\mathrm{d}_{\mathrm{req}}{ }^{2}=\frac{M u}{\Phi \text { fy b } \rho\left(1-.59 \rho \frac{f y}{f^{\prime} c}\right)}=\frac{11.78 * 12}{.9 * 60 * 12 * .015 *\left(1-.59 * .015 * \frac{60}{3}\right)}$

$$
\text { So, } d_{\text {req }}=4.20^{\prime \prime}<7^{\prime \prime}(O K)
$$

$$
\mathrm{A}_{\mathrm{s}, \min }=.0018 \mathrm{bt}=.0018 * 12 * 8=0.173 \mathrm{in}^{2} / \mathrm{ft}=111.61 \mathrm{~mm}^{2}
$$

$$
\begin{aligned}
& \text { Now, }+\mathrm{A}_{\mathrm{s}}=\frac{+M}{\phi f y\left(d-\frac{a}{2}\right)} \quad ; \mathrm{a}=\frac{A s f y}{.85 f^{\prime} c b} \\
&=\frac{7 * 12}{.9 * 60 *\left(7-\frac{.45}{2}\right)}=\frac{0.23 * 60}{.85 * 3 * 12}=0.45^{\prime \prime} \\
&=0.230 \mathrm{in}^{2} / \mathrm{ft}=148.13 \mathrm{~mm}^{2}>\mathrm{A}_{\mathrm{s}, \min }(\mathrm{OK})
\end{aligned}
$$

Using $\Phi 10 \mathrm{~mm}$, Area $=78.5 \mathrm{~mm}^{2}$, spacings $=\frac{78.5 x 12}{148.13}=6.35^{\prime \prime}=6$ " [ not more than 2 d$]$ (OK)

$$
\phi 10 \mathrm{~mm} @ 6^{\prime \prime} \mathrm{c} / \mathrm{c} \text { alt. ckd. at bottom }
$$

$$
\text { Again, } \begin{aligned}
-\mathrm{A}_{\mathrm{s}, \text { interior }} & =\frac{-M}{\phi f y\left(d-\frac{a}{2}\right)} \quad ; \mathrm{a}=\frac{A s f y}{.85 f^{\prime} c b} \\
& =\frac{11.78 * 12}{.9 * 60 *\left(6.5-\frac{.78}{2}\right)} \\
& =0.40 \mathrm{in}^{2} / \mathrm{ft}=258.064 \mathrm{~mm}^{2}
\end{aligned} \quad=\frac{.417 * 60}{.85 * 3 * 12}=0.78^{\prime \prime}
$$

So, using $\Phi 10 \mathrm{~mm}$, distance between ckd bars $6 \times 2=12^{\prime \prime}$
Provided Steel $=\frac{12 x 78.5}{12}=78.5 \mathrm{~mm}^{2} ;$ So. required steel $=258.064-78.5=179.56 \mathrm{~mm}^{2}$
Using $3 Ф 10 \mathrm{~mm}$ as extra top ,area $=3 \times 78.5=235.5 \mathrm{~mm}^{2}>179.56 \mathrm{~mm}^{2}$

## 3- $\$ 10 \mathrm{~mm}$ extra top betweenckd. bars at interior end

$$
\text { Again, } \begin{aligned}
-\mathrm{A}_{\mathrm{s}, \text { exterior }} & =\frac{-M}{\phi f y\left(d-\frac{a}{2}\right)} \quad ; \mathrm{a}=\frac{A s f y}{.85 f^{\prime} c b} \\
& =\frac{5.83 * 12}{.9 * 60 *\left(6.5-\frac{.373}{2}\right)} \\
& =0.19 \mathrm{in}^{2} / \mathrm{ft}=122.58 \mathrm{~mm}^{2}>\mathrm{A}_{\mathrm{s}, \min }(\mathrm{OK})
\end{aligned} \quad=\frac{. .19 * 60}{.85 * 3 * 12}=0.373^{\prime \prime}
$$

So, using $\Phi 10 \mathrm{~mm}$, distance between ckd bars $6 \times 2=12^{\prime \prime}$
Provided Steel $=\frac{12 x 78.5}{12}=78.5 \mathrm{~mm}^{2} ;$ So. required steel $=122.58-78.5=44.08 \mathrm{~mm}^{2}$
Using $1 \Phi 10 \mathrm{~mm}$ as extra top ,area=1x78.5 $=78.5 \mathrm{~mm}^{2}>44.08 \mathrm{~mm}^{2}$
$1-\$ 10 \mathrm{~mm}$ extra top between ckd. Bars at exterior end
(iv) Middle strip:

Width of middle strip $=20 / 2+18 / 2-9.5=9.5 \mathrm{ft}$
$\mathrm{M}_{\text {-ve, interior }}=37.3 / 9.5=3.93 \mathrm{k}-\mathrm{ft} / \mathrm{ft}$
$M_{\text {+ve }}=44.32 / 9.5=4.67 \mathrm{k}-\mathrm{ft} / \mathrm{ft}$

$$
\mathrm{A}_{\mathrm{s}, \text { min }}=.0018 \mathrm{bt}=.0018 * 12 * 8=0.173 \mathrm{in}^{2} / \mathrm{ft}=111.61 \mathrm{~mm}^{2}
$$

Now, $+\mathrm{A}_{\mathrm{s}}=\frac{+M}{\phi f y\left(d-\frac{a}{2}\right)} \quad ; \mathrm{a}=\frac{A s f y}{.85 f^{\prime} c b}$

$$
\begin{aligned}
& =\frac{4.67 * 12}{.9 * 60 *\left(6.5-\frac{.297}{2}\right)} \\
& =0.151 \quad \mathrm{in}^{2} / \mathrm{ft}=97.42 \mathrm{~mm}^{2}<\mathrm{A}_{\mathrm{s}, \text { min }}
\end{aligned} \quad=\frac{0.151 * 60}{.85 * 3 * 12}=0.297^{\prime \prime}
$$

So, $A_{s, \min }=111.61 \mathrm{~mm}^{2}$ governs.
Using $\Phi 10 \mathrm{~mm}$, Area $=78.5 \mathrm{~mm}^{2}$, spacings $=\frac{78.5 \times 12}{111.61}=8.44=8 "[$ not more than 2 d$]$ (OK)

## $\phi 10 \mathrm{~mm}$ @ 8 " c/c alt. ckd. at bottom

$$
\text { Again, } \begin{aligned}
-\mathrm{A}_{\mathrm{s}, \text { interior }} & =\frac{-M}{\phi f y\left(d-\frac{a}{2}\right)} \quad ; \mathrm{a}=\frac{A s f y}{.85 f^{\prime} c b} \\
& =\frac{3.93 * 12}{.9 * 60 *\left(6.5-\frac{.25}{2}\right)} \\
& =0.13 \mathrm{in}^{2} / \mathrm{ft}=83.87 \mathrm{~mm}^{2}
\end{aligned} \quad=\frac{.13 * 60}{.85 * 3 * 12}=0.25^{\prime \prime}
$$

Distance between two cranked bars $=8 * 2=16 "$
Provided Steel $=\frac{12 x 78.5}{16}=58.87 \mathrm{~mm}^{2}$;So. required steel $=83.87-58.87=25 \mathrm{~mm}^{2}$
Using $1 \Phi 10 \mathrm{~mm}$ as extra top ,area $=1 \times 78.5=78.5 \mathrm{~mm}^{2}>25 \mathrm{~mm}^{2}$

1- $\$ 10 \mathrm{~mm}$ extra top between ckd. bars at interior end


Fig 3.13: Reinforcement Details

### 3.2.4 Sample calculation of an exterior frame (without edge beam) from excel spreadsheet (using DDM)



| Thickness (in) | 8 |
| :--- | :---: |
| Effective depth, d (in) | 7 |
| DL (ksf) | 0.125 |
| LL (ksf) | 0.04 |
| Wu (ksf) | 0.214 |
| Contributed punching area (sft) | 397.4931 |
| Vp (k) | 85.06351 |
| bO (in) | 76 |
| $\phi$ Vc (k) | 87.41652 |
| Mo (k-ft) | 193.135 |
| Int neg M (k-ft) | 135.1945 |
| Pos M (k-ft) | 50.4302 |
| Ext neg M (k-ft) | 50.2151 |
| Ext neg M @ Col strp (k-ft) | 0 |
| Ext neg M @ mid strp (k-ft) | 60.25812 |
| Pos M @ Col strp (k-ft) | 40.17208 |
| Pos M @ mid strp (k-ft) | 101.3959 |
| Int neg M @ Col strp (k-ft) | 33.79863 |
| Int neg M @ mid strp (k-ft) | 0.015482 |
| $\rho$ max | 0.0018 |
| $\rho$ min | 0.1728 |
| As min (in2/ft) | 14 |
| Max spacing (in) |  |


| Pos rebar @ col strp |  |
| :--- | ---: |
| As (for a=1in) (in2/ft) | 0.206011 |
| a (in) | 0.403942 |
| As (trial 2) (in2/ft) | 0.196979 |
| a (in) | 0.386233 |
| As (trial 3) (in2/ft) | 0.196723 |
| As (control) (in2/ft) | 0.196723 |
| Spacing (in) | 10 |
| Spacing (control) (in) | 10 |


| Pos rebar @ mid strp |  |
| :--- | ---: |
| As (for a=1in) (in2/ft) | 0.13734 |
| a (in) | 0.269295 |
| As (trial 2) (in2/ft) | 0.130032 |
| a (in) | 0.254964 |
| As (trial 3) (in2/ft) | 0.129896 |
| As (control) (in2/ft) | 0.1728 |
| Spacing (in) | 12 |
| Spacing (control) (in) | 12 |


| Neg ext rebar @ col strp |  |
| :--- | ---: |
| As (for a=1in) (in2/ft) | 0.171676 |
| a (in) | 0.336619 |
| As (trial 2) (in2/ft) | 0.16334 |
| a (in) | 0.320275 |
| As (trial 3) (in2/ft) | 0.163145 |
| As (control) (in2/ft) | 0.163145 |
| Spacing (in) | 12 |
| Spacing (control) (in) | 12 |


| Neg ext rebar @ mid strp |  |
| :--- | :--- |
| As (for a=1in) (in2/ft) |  |
| a (in) |  |
| As (trial 2) (in2/ft) |  |
| a (in) |  |
| As (trial 3) (in2/ft) |  |
| As (control) (in2/ft) |  |
| Spacing (in) |  |
| Spacing (control) (in) |  |

Neg int rebar @ col strp

| As (for a=1in) (in2/ft) | 0.346653 |
| :--- | ---: |
| $a(i n)$ | 0.679711 |
| As (trial 2) (in2/ft) | 0.338317 |
| $a$ (in) | 0.663367 |
| As (trial 3) (in2/ft) | 0.337903 |
| As (control) (in2/ft) | 0.337903 |
| Spacing (in) | 6 |
| Spacing (control) (in) | 6 |

Neg int rebar @ mid strp

| As (for a=1in) (in2/ft) | 0.115551 |
| :--- | ---: |
| a (in) | 0.22657 |
| As (trial 2) (in2/ft) | 0.109062 |
| a (in) | 0.213848 |
| As (trial 3) (in2/ft) | 0.108962 |
| As (control) (in2/ft) | 0.108962 |
| Spacing (in) | 19 |
| Spacing (control) (in) | 14 |


| Total weight of steel | 187.4337 kg |
| :---: | :--- | :--- |
| Total volume of concrete | 273.3333 cft |

### 3.2.5 Sample calculation of an interior frame from excel spreadsheet (using DDM)



| Thickness (in) | 8 |
| :--- | :---: |
| Effective depth, d (in) | 7 |
| DL (ksf) | 0.125 |
| LL (ksf) | 0.04 |
| Wu (ksf) | 0.214 |
| Contributed punching area (sft) | 397.4931 |
| Vp (k) | 85.06351 |
| bO (in) | 76 |
| $\phi$ Vc (k) | 87.41652 |
| Mo (k-ft) | 193.135 |
| Pos M (k-ft) | 67.59725 |
| Neg M (k-ft) | 125.5378 |
| Pos M @ Col strp (k-ft) | 40.55835 |
| Neg M @ Col strp (k-ft) | 94.15331 |
| Pos M @ mid strp (k-ft) | 27.0389 |
| Neg M @ mid strp (k-ft) | 31.38444 |
| $\rho$ max | 0.015482 |
| $\rho$ min | 0.0018 |
| As min (in2/ft) | 0.1728 |
| Max spacing (in) | 14 |


| Pos rebar @ col strp |  |
| :--- | ---: |
| As (for a=1in) (in2/ft) | 0.138661 |
| a (in) | 0.271884 |
| As (trial 2) (in2/ft) | 0.131307 |
| a (in) | 0.257464 |
| As (trial 3) (in2/ft) | 0.131169 |
| As (control) (in2/ft) | 0.1728 |
| Spacing (in) | 12 |
| Spacing (control) (in) | 12 |

Pos rebar @ mid strp

| As (for a $=1 \mathrm{in})$ (in2/ft) | 0.092441 |
| :--- | ---: |
| a (in) | 0.181256 |
| As (trial 2) (in2/ft) | 0.086964 |
| a (in) | 0.170517 |
| As (trial 3) (in2/ft) | 0.086896 |
| As (control) (in2/ft) | 0.1728 |
| Spacing (in) | 12 |
| Spacing (control) (in) | 12 |


| Neg rebar @ col strp |  |
| :--- | ---: |
| As (for a=1in) (in2/ft) | 0.321892 |
| a (in) | 0.63116 |
| As (trial 2) (in2/ft) | 0.313011 |
| a (in) | 0.613747 |
| As (trial 3) (in2/ft) | 0.312604 |
| As (control) (in2/ft) | 0.312604 |
| Spacing (in) | 6 |
| Spacing (control) (in) | 6 |


| Neg ext rebar @ mid strp |  |
| :--- | ---: |
| As (for a=1in) (in2/ft) | 0.107297 |
| a (in) | 0.210387 |
| As (trial 2) (in2/ft) | 0.101153 |
| a (in) | 0.19834 |
| As (trial 3) (in2/ft) | 0.101065 |
| As (control) (in2/ft) | 0.101065 |
| Spacing (in) | 20 |
| Spacing (control) (in) | 14 |


| Total weight of steel | 203.8088 kg |
| :---: | :--- |
| Total volume of concrete | 266.6667 cft |

### 3.3 Beam-supported slab

Step 1: At first, we studied how to design a beam-supported slab by Direct Design Method, from text-books and ACI codes.

Step 2: We designed one interior frame and one exterior frame, by hand calculation, to get familiar with the design procedure.

Step 3: After understanding the design procedure properly, we started to formulate excel spreadsheets, based on what we learnt on previous two steps. Two separate spreadsheets were formulated to design both interior and exterior frames. Those were prepared in way that it would take c/c span lengths of a frame, largest spans along both axes of the floor, column cross-sections, slab-thickness, and different design data as inputs; and would give outputs: slab thickness, bending-moments in both column and middle strips, rebar detailing (showing rebar diameter, spacing and cut-off lengths), width of column and middle strips, and estimated materials (weight of reinforcing steel and volume of concrete).

Step 4: As per table 3.1, inputs were put in the spreadsheets to design and estimate the floors. The first task to work with the spreadsheets was the determination of the slab thickness, which were based on three criteria:

- Minimum thickness to control deflections, established by ACI code 9.5.3.3
- Minimum effective depth for maximum reinforcement ratios, established by ACI code 10.3.5
- Minimum thickness to satisfy ACI code 9.5.3.3(c)

The worksheets were formulated in such a way that users not only can input the slab thickness, but also see suggested thicknesses satisfying those three conditions (based on the input thickness). So, at first, an assumed thickness had been input in the worksheets for the frame having the largest clear span of the floor, and then observing the other suggestions, input thickness was being changed simultaneously. At the last stage, a thickness had been input that satisfied all the criteria, and not exceeding them. Then that thickness was put in the worksheet for all the frames of a same floor.

Step 5: After designing all the frames and estimation of a floor, estimated material quantities from outputs of those spreadsheets, were summed to estimate total material quantity for the designed floor-system. By the same approach, all the floors were estimated shown in table 3.1.

Step 6: Using all the estimated material quantities for different floors, were used to calculate costs. For cost analysis, the considered types of costs were assumed same as flat-plate. Those three types of costs were summed and expressed as per square-feet of floor area, which were the total cost for a certain floor.

Step 7: Using the total material quantities and total costs, different column-charts had been developed to interpret cost analysis. Those charts are shown in chapter 5.

### 3.3.1 Sample calculation of an exterior frame from excel spreadsheet (using DDM)



| Thickness (in) | 6 |
| :--- | :---: |
| Effective depth, d (in) | 5 |
| DL (ksf) | 0.1 |
| LL (ksf) | 0.04 |
| Wu (ksf) | 0.184 |
| Mo (k-ft) | 166.06 |
| Int neg M (k-ft) | 116.242 |
| Pos M (k-ft) | 94.6542 |
| Ext neg M (k-ft) | 26.5696 |
| Ext neg M @ Col strp (k-ft) | 3.830207 |
| Ext neg M @ mid strp (k-ft) | 1.034886 |
| Pos M @ Col strp (k-ft) | 10.6486 |
| Pos M @ mid strp (k-ft) | 23.66355 |
| Int neg M @ Col strp (k-ft) | 13.07723 |
| Int neg M @ mid strp (k-ft) | 29.0605 |
| $\rho$ max | 0.015482 |
| $\rho$ min | 0.0018 |
| As min (in2/ft) | 0.1296 |
| Max spacing (in) | 10 |


| (a) |  | (b) |  |
| :---: | :---: | :---: | :---: |
| x1 (in) | 6 | x1 (in) | 12 |
| y1 (in) | 19 | y1 (in) | 13 |
| x2 (in) | 7 | x2 (in) | 6 |
| y2 (in) | 12 | y2 (in) | 7 |
| c1 (in4) | 1095.84 | c1 (in4) | 3133.44 |
| c2 (in4) | 867.79 | c2 (in4) | 231.84 |
| C (in4) | 1963.63 | C (in4) | 3365.28 |
|  |  |  |  |
| C max (in4) | 3365.28 |  |  |
| Ib (in4) | 4394 |  |  |
| Is (in4) | 4320 |  |  |
| $\alpha$ | 1.01713 |  |  |
| $\beta \mathrm{t}$ | 0.3895 |  |  |
| corrected $\beta$ t | 0.3895 |  |  |
| L2/L1 | 1 | \% Col ext - | 96.105 |
| $\alpha^{*}$ L2/L1 | 1.01713 | \% Col int - | 75 |
| corrected $\alpha^{*}$ L2/L1 | 1 | \% Col + | 75 |
|  |  |  |  |

Pos rebar @ col strp

| As (for a=1in) (in2/ft) | 0.052586 |
| :--- | ---: |
| a (in) | 0.103109 |
| As (trial 2) (in2/ft) | 0.04782 |
| a (in) | 0.093765 |
| As (trial 3) (in2/ft) | 0.047775 |
| As (control) (in2/ft) | 0.1296 |
| Spacing (in) | 16 |
| Spacing (control) (in) | 10 |

Pos rebar @ mid strp

| As (for a=1in) (in2/ft) | 0.116857 |
| :--- | ---: |
| a (in) | 0.229131 |
| As (trial 2) (in2/ft) | 0.107638 |
| a (in) | 0.211054 |
| As (trial 3) (in2/ft) | 0.107439 |
| As (control) (in2/ft) | 0.1296 |
| Spacing (in) | 16 |
| Spacing (control) (in) | 10 |


| Neg ext rebar @ col strp |  |
| :--- | ---: |
| As (for a=1in) (in2/ft) | 0.018915 |
| a (in) | 0.037087 |
| As (trial 2) (in2/ft) | 0.017087 |
| a (in) | 0.033503 |
| As (trial 3) (in2/ft) | 0.01708 |
| As (control) (in2/ft) | 0.01708 |
| Spacing (in) | 122 |
| Spacing (control) (in) | 10 |

## Neg ext rebar @ mid strp

| As (for a=1in) (in2/ft) | 0.005111 |
| :--- | ---: |
| a (in) | 0.010021 |
| As (trial 2) (in2/ft) | 0.004604 |
| a (in) | 0.009028 |
| As (trial 3) (in2/ft) | 0.004604 |
| As (control) (in2/ft) | 0.004604 |
| Spacing (in) | 456 |
| Spacing (control) (in) | 10 |

Neg int rebar @ col strp

| As (for a=1in) (in2/ft) | 0.064579 |
| :--- | ---: |
| a (in) | 0.126625 |
| As (trial 2) (in2/ft) | 0.058866 |
| a (in) | 0.115424 |
| As (trial 3) (in2/ft) | 0.0588 |
| As (control) (in2/ft) | 0.0588 |
| Spacing (in) | 35 |
| Spacing (control) (in) | 10 |


| Neg int rebar @ mid strp |  |
| :--- | ---: |
| As (for a=1in) (in2/ft) | 0.143509 |
| a (in) | 0.281389 |
| As (trial 2) (in2/ft) | 0.132897 |
| a (in) | 0.260583 |
| As (trial 3) (in2/ft) | 0.132613 |
| As (control) (in2/ft) | 0.132613 |
| Spacing (in) | 15 |
| Spacing (control) (in) | 10 |


| Total weight of steel |  |  | 221.6048 |
| :--- | ---: | :--- | :--- |
| Total vg |  |  |  |
| Slab | 205 | cft |  |
| Beam | 11.08333 | cft |  |

### 3.3.2 Sample calculation of an interior frame from excel spreadsheet (using DDM)



| Thickness (in) | 6 | Ib (in4) | 4394 |
| :---: | :---: | :---: | :---: |
| Effective depth, d (in) | 5 | Is (in4) | 4320 |
| DL (ksf) | 0.1 | $\alpha$ | 1.01713 |
| LL (ksf) | 0.04 | L2/L1 | 1 |
| Wu (ksf) | 0.184 | 人*L2/L1 | 1.01713 |
| Mo (k-ft) | 166.06 | corrected $\alpha^{*}$ L2/L1 | 1 |
| Pos M (k-ft) | 58.121 |  |  |
| Neg M (k-ft) | 107.939 | \% Col - | 75 |
| Pos M @ Col strp (k-ft) | 6.538613 | \% Col + | 75 |
| Neg M @ Col strp (k-ft) | 12.14314 |  |  |
| Pos M @ mid strp (k-ft) | 14.53025 |  |  |
| Neg M @ mid strp (k-ft) | 26.98475 |  |  |
| $\rho$ max | 0.015482 |  |  |
| $\rho$ min | 0.0018 |  |  |
| As min (in2/ft) | 0.1296 |  |  |
| Max spacing (in) | 10 |  |  |

Pos rebar @ col strp

| As (for a=1in) (in2/ft) | 0.032289 |
| :--- | ---: |
| a (in) | 0.063313 |
| As (trial 2) (in2/ft) | 0.029246 |
| a (in) | 0.057344 |
| As (trial 3) (in2/ft) | 0.029228 |
| As (control) (in2/ft) | 0.1296 |
| Spacing (in) | 16 |
| Spacing (control) (in) | 10 |

## Pos rebar @ mid strp

| As (for a=1in) (in2/ft) | 0.071754 |
| :--- | ---: |
| a (in) | 0.140695 |
| As (trial 2) (in2/ft) | 0.0655 |
| a (in) | 0.128432 |
| As (trial 3) (in2/ft) | 0.065419 |
| As (control) (in2/ft) | 0.1296 |
| Spacing (in) | 16 |
| Spacing (control) (in) | 10 |

Neg rebar @ col strp

| As (for a=1in) (in2/ft) | 0.059966 |
| :--- | ---: |
| a (in) | 0.117581 |
| As (trial 2) (in2/ft) | 0.054612 |
| a (in) | 0.107082 |
| As (trial 3) (in2/ft) | 0.054554 |
| As (control) (in2/ft) | 0.054554 |
| Spacing (in) | 38 |
| Spacing (control) (in) | 10 |

Neg ext rebar @ mid strp

| As (for a=1in) (in2/ft) | 0.133258 |
| :--- | ---: |
| a (in) | 0.26129 |
| As (trial 2) (in2/ft) | 0.12315 |
| a (in) | 0.241471 |
| As (trial 3) (in2/ft) | 0.1229 |
| As (control) (in2/ft) | 0.1229 |
| Spacing (in) | 17 |
| Spacing (control) (in) | 10 |


| Total weight of steel |  |  | 226.6798 |  | kg |
| :--- | :--- | :--- | :--- | :---: | :---: |
| Total volume of concrete |  |  |  |  |  |
| Slab | 200 | ctt |  |  |  |
| Beam | 11.08333 cft |  |  |  |  |

### 3.4 Design Example of two way beam supported slab (using Moment coefficient method)

Beam-column supported floor slab of a 93 'x75' (center to center distance of extreme columns) is to carry service live load of 100 psf in addition to its own weight, $1 / 2^{\prime \prime}$ thick plaster and $3 / 2^{\prime \prime}$ thick floor finish. Supporting columns of 12in square are spaced orthogonally at an interval at 31 ' and 25 ' on centers along longitudinal and transverse directions respectively. Width of each beam is 12 in . Using BNBC/ACl code of moment coefficients design the slab by USD method, if $f^{\prime}=3000$ psi and $f_{y}=60000$ psi.
$3 @ 31^{\prime}=93^{\prime}$

3 @ 25'
$=75^{\prime}$


Figure 03: Slab panel orientation and case type, e.g., case 9 is typical exterior, 4 is corner slab etc.

Here $A=25^{\prime}-11^{\prime}=24^{\prime}$ and $B=31^{\prime}-1^{\prime}=30^{\prime}=I_{n}$.
$\mathrm{t}=\frac{l_{n}\left(0.8+\left(\frac{f_{y}}{200000}\right)\right)}{36+9 \beta}=\frac{30 *\left(0.8+\left(\frac{60000}{200000}\right)\right)}{36+9 * \frac{* 0}{24}}=8.38 " \approx 8.5^{\prime \prime}$ say.
So $d=8.5^{\prime \prime}-1$ " $=7.5^{\prime}$
$W_{D L}=(8.5+0.5+1.5) * 12.5 * 1.2=157.5 \mathrm{psf}$

| $\mathrm{W}_{\mathrm{LL}}=$ | $100 * 1.6=160 \mathrm{psf}$ |
| :--- | ---: |
| $\mathrm{W}_{\mathrm{u}}$ | $=317.5 \mathrm{psf}$ |

```
m=A/B=24/30=0.8
```

|  | 2 | 4 | 8 | 9 |
| :--- | :--- | :--- | :--- | :--- |
| $-C_{A}$ | 0.065 | 0.071 | 0.055 | 0.075 |
| $-C_{B}$ | 0.027 | 0.029 | 0.041 | 0.017 |
| $C_{A D L}$ | 0.026 | 0.039 | 0.032 | 0.029 |
| $C_{B L L}$ | 0.011 | 0.016 | 0.015 | 0.010 |
| $C_{A L L}$ | 0.041 | 0.048 | 0.044 | 0.042 |
| $C_{B}$ | 0.020 | 0.019 | 0.017 |  |

[Note: In this slab, there are four different types of cases among all panels. We take the maximum value of moment coefficient from four cases.]

$$
\begin{aligned}
+\mathrm{M}_{\mathrm{A}} & =\mathrm{C}_{\mathrm{ADL}} * \mathrm{~W}_{\mathrm{DL}} * \mathrm{~A}^{2}+\mathrm{C}_{\mathrm{ALL}} * \mathrm{WLL}_{\mathrm{LL}} \mathrm{~A}^{2} \\
& =0.039 * 157.5^{*} 24^{2}+0.048^{*} 160 * 24^{2} \\
& =7961.76 \mathrm{lb}-\mathrm{ft} / \mathrm{ft} \\
& =7.96 \mathrm{k}-\mathrm{ft} / \mathrm{ft} \\
-\mathrm{M}_{\mathrm{A}} & =\mathrm{C}_{\mathrm{A}} * \mathrm{~W}_{\mathrm{u}} * \mathrm{~A}^{2} \\
& =0.075 * 317.5^{*} 24^{2} \\
& =13716 \mathrm{lb}-\mathrm{ft} / \mathrm{ft} \\
& =13.6 \mathrm{k}-\mathrm{ft} / \mathrm{ft} \\
+\mathrm{M}_{\mathrm{B}} & =\mathrm{C}_{\mathrm{B}} \mathrm{DL}^{*} \mathrm{~W}_{\mathrm{DL}} \mathrm{~B}^{2}+\mathrm{C}_{\mathrm{B}} \mathrm{LL} * \mathrm{~W}_{\mathrm{LL}} * \mathrm{~B}^{2} \\
& =0.016^{*} 157.5^{*} 30^{2}+0.020^{*} 160^{*} 30^{2} \\
& =5148 \mathrm{lb}-\mathrm{ft} / \mathrm{ft} \\
& =5.148 \mathrm{k}-\mathrm{ft} / \mathrm{ft} \\
-\mathrm{M}_{\mathrm{B}} & =\mathrm{C}_{\mathrm{B}} * \mathrm{~W}_{\mathrm{u}} * \mathrm{~B}^{2} \\
& =0.041 * 317.5^{*} 30^{2} \\
& =11716 \mathrm{lb}-\mathrm{ft} / \mathrm{ft} \\
& =11.716 \mathrm{k}-\mathrm{ft} / \mathrm{ft}
\end{aligned}
$$

Rebar for short direction/transverse direction:
$+\mathrm{A}_{\mathrm{SA}}=\frac{\mathrm{M} * 12}{0.9 * \mathrm{f}_{\mathrm{y}} *\left(\mathrm{~d}-\frac{\mathrm{a}}{2}\right)}=\frac{\mathrm{M} * 12}{0.9 * 60 *\left(\mathrm{~d}-\frac{\mathrm{a}}{2}\right)}=\frac{\mathrm{M}}{4.5 *\left(\mathrm{~d}-\frac{\mathrm{a}}{2}\right)}=\frac{7.96}{4.5 *(7.5-0.24)}=0.244 \mathrm{in}^{2} / \mathrm{ft}$ (Controlling).
and $\mathrm{a}=\frac{\mathrm{A}_{\mathrm{s}} \mathrm{f} \mathrm{y}}{0.85 f_{\mathrm{c}}^{\prime} \mathrm{b}}=\frac{\mathrm{A}_{\mathrm{s}} * 60}{0.85 * 3 * 12}=1.96 * \mathrm{~A}_{\mathrm{s}}=1.96 * 0.244=0.478 \mathrm{in}$.
$A_{\text {min }}=0.0018 \times b x t=0.0018 \times 12 \times 8.5=0.1836 \mathrm{in}^{2} / \mathrm{ft}$
Using $\phi 10 \mathrm{~mm}$ bar
$\mathrm{S}=\frac{\text { Area of bar used } * \text { width of strip }}{\text { Requried } \mathrm{A}_{\mathrm{s}}}=\frac{0.121 * 12}{0.244}=5.95^{\prime \prime} \approx 5.5 \mathrm{c} \mathrm{c} / \mathrm{c}$ at bottom along short direction crank 50\% bar to negative zone.
$-\mathrm{A}_{\mathrm{SA}}=\frac{\mathrm{M}}{4.5 *\left(\mathrm{~d}-\frac{\mathrm{a}}{2}\right)}=\frac{13.61}{4.5 *(7.5-0.42)}=0.427 \mathrm{in}^{2} / \mathrm{ft}$ (Controlling).
$\mathrm{a}=1.96 * \mathrm{~A}_{\mathrm{s}}=0.838$ in
$A_{\text {min }}=0.1836 \mathrm{in}^{2} / \mathrm{ft}$.
Already provided $\mathrm{A}_{\mathrm{s} 1}=\frac{0.121 * 12}{11}=0.132 \mathrm{in}^{2} / \mathrm{ft}$
Extra top required, $\mathrm{A}_{52}=(0.427-0.132)=0.295 \mathrm{in}^{2} / \mathrm{ft}$.
Using $\Phi 10 \mathrm{~mm}$ bar $\mathrm{S}=4.92 \approx 4.5^{\mathrm{\prime}} \mathrm{c} / \mathrm{c}$ extra top.
Rebar along long direction:
$+\mathrm{A}_{\mathrm{S}}=\frac{5.148}{4.5 *(7.5-0.15)}=0.155 \mathrm{in}^{2} / \mathrm{ft}$
$A_{\text {min }}=0.1836 \mathrm{in}^{2} / \mathrm{ft}$ (Controlling).
Using $\Phi 10 \mathrm{~mm}$ bar @ 7.90 " $\approx 7.5^{\prime \prime} \mathrm{c} / \mathrm{c}$ at bottom along long direction crank $50 \%$ bar to negative zone.
$-A_{S B}=\frac{11.716}{4.5 *(7.5-0.36)}=0.365 \mathrm{in}^{2} / \mathrm{ft}$
Already provided $\mathrm{A}_{\mathrm{s} 1}=\frac{0.121 * 12}{15}=0.0968 \mathrm{in}^{2} / \mathrm{ft}$
Extra top required, $\mathrm{A}_{\mathrm{s} 2}=(0.365-00.0968) \mathrm{in}^{2} / \mathrm{ft}=0.2682 \mathrm{in}^{2} / \mathrm{ft}$
Using © 10mm bar @ 5.41" $\sim 5$ " c/c extra top


Fig 3.14: Reinforcement details of slab in plan.

Chapter 4
RESULTS \&
DISCUSSION

The comparative analysis, based on the designed and estimated floor-systems as per table 3.1, was done in a way described in chapter 3 . The final comparisons had been summarized in some tables, which are shown in tables 4.1, 4.2, 4.3, 4.4 and 4.5.

Table 4.1: Comparison table for long to short span ratio, $\beta=1.00$

| Long span (ft) |  | 10 | 15 | 20 | 25 | 30 | 35 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Short span (ft) |  | 10 | 15 | 20 | 25 | 30 | 35 |
|  | Thickness (in) | 5 | 6 | 8 | 12.5 | 18.5 | 25.5 |
|  | Steel (kg) | 1228.53 | 2142.56 | 3528.9 | 8000.24 | 15295.4 | 31349.2 |
|  | Concrete (cft) | 375 | 1012.5 | 2400 | 5859.38 | 12487.5 | 23428.13 |
|  | Steel (BDT in thousand) | 61.4264 | 107.1282 | 176.4448 | 400.0118 | 764.77 | 1567.46 |
|  | Concrete (BDT in thousand) | 93.75 | 253.125 | 600 | 1464.844 | 3121.875 | 5857.033 |
|  | Formwork (BDT in thousand) | 16.65 | 44.955 | 106.56 | 260.1563 | 554.445 | 1040.209 |
|  | Labor (BDT in thousand) | 0.9 | 2.025 | 3.6 | 5.625 | 8.1 | 11.025 |
|  | Total cost (BDT/sft) | 191.9182 | 201.1028 | 246.2791 | 378.7799 | 549.2827 | 768.7734 |
| $\frac{0}{0}$$\frac{0}{0}$00000000 | Thickness (in) | 3.5 | 4.5 | 6 | 7.5 | 9 | 10.5 |
|  | Steel (kg) | 2077.61 | 3376.78 | 4072.94 | 5435.45 | 8242.21 | 13389.64 |
|  | Concrete (cft) | 325.5 | 913.37 | 2066.01 | 3971.63 | 6712.99 | 10496.96 |
|  | Steel (BDT in thousand) | 103.8804 | 168.839 | 203.6468 | 271.7724 | 412.1104 | 669.482 |
|  | Concrete (BDT in thousand) | 81.375 | 228.3428 | 516.501 | 992.9063 | 1678.248 | 2624.24 |
|  | Formwork (BDT in thousand) | 14.4522 | 40.55367 | 91.73058 | 176.3402 | 298.0568 | 466.065 |
|  | Labor (BDT in thousand) | 0.9 | 2.025 | 3.6 | 5.625 | 8.1 | 11.025 |
|  | Total cost (BDT/sft) | 222.8973 | 217.1656 | 226.5218 | 257.1811 | 295.8661 | 342.0238 |

Table 4.2: Comparison table for long to short span ratio, $\beta=1.25$

| Long span (ft) |  | 12.5 | 18.75 | 25 | 31.25 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Short span (ft) |  | 10 | 15 | 20 | 25 |
| $\begin{aligned} & \mathbb{\#} \\ & \frac{\pi}{O} \\ & \frac{\pi}{\square} \end{aligned}$ | Thickness (in) | 5 | 12 | 12 | 18 |
|  | Steel (kg) | 1507.14 | 2911.26 | 5583.76 | 11873.84 |
|  | Concrete (cft) | 468.75 | 2531.25 | 4500 | 10546.88 |
|  | Steel (BDT in thousand) | 75.357 | 145.563 | 279.188 | 593.692 |
|  | Concrete (BDT in thousand) | 117.1875 | 632.8125 | 1125 | 2636.72 |
|  | Form-work (BDT in thousand) | 20.8125 | 112.3875 | 199.8 | 468.28147 |
|  | Labor (BDT in thousand) | 1.125 | 2.5313 | 4.5 | 7.0313 |
|  | Total cost (BDT/sft) | 190.6507 | 352.9064 | 357.4418 | 527.0364 |
|  | Thickness (in) | 3.5 | 5.5 | 7 | 9 |
|  | Steel (kg) | 2570.51 | 3217.04 | 4228.68 | 6980.58 |
|  | Concrete (cft) | 422.87 | 1356.36 | 2979 | 5828.44 |
|  | Steel (BDT in thousand) | 128.5252 | 160.852 | 211.434 | 349.029 |
|  | Concrete (BDT in thousand) | 105.7175 | 339.089 | 744.75 | 1457.11 |
|  | Form-work (BDT in thousand) | 18.7754 | 60.2222 | 132.2676 | 258.7827 |
|  | Labor (BDT in thousand) | 1.125 | 2.5313 | 4.5 | 7.0313 |
|  | Total cost (BDT/sft) | 225.905 | 222.299 | 242.8781 | 294.6778 |

Table 4.3: Comparison table for long to short span ratio, $\beta=1.50$

| Long span (ft) |  | 15 | 22.5 | 30 |
| :---: | :---: | :---: | :---: | :---: |
| Short span (ft) |  | 10 | 15 | 20 |
|  | Thickness (in) | 6 | 9 | 12 |
|  | Steel (kg) | 1391.72 | 3269.88 | 7603.9 |
|  | Concrete (cft) | 675 | 2278.13 | 5400 |
|  | Steel (BDT in thousand) | 69.5858 | 163.4942 | 380.195 |
|  | Concrete (BDT in thousand) | 168.75 | 569.5313 | 1350 |
|  | Form-work (BDT in thousand) | 29.97 | 101.1488 | 239.76 |
|  | Labor (BDT in thousand) | 1.35 | 3.0375 | 5.4 |
|  | Total cost (BDT/sft) | 199.745 | 275.6253 | 365.8065 |
| $\begin{aligned} & \frac{0}{0} \\ & \frac{0}{N} \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | Thickness (in) | 4 | 6 | 8 |
|  | Steel (kg) | 2575.39 | 3418.47 | 4781.07 |
|  | Concrete (cft) | 561 | 1768.35 | 4043.04 |
|  | Steel (BDT in thousand) | 128.7693 | 170.9234 | 239.0534 |
|  | Concrete (BDT in thousand) | 140.25 | 442.0875 | 1010.76 |
|  | Form-work (BDT in thousand) | 24.9084 | 78.5147 | 179.511 |
|  | Labor (BDT in thousand) | 1.35 | 3.0375 | 5.4 |
|  | Total cost (BDT/sft) | 218.7242 | 228.6628 | 265.6897 |

Table 4.4: Comparison table for long to short span ratio, $\beta=1.75$

| Long span (ft) |  | 17.5 | 26.25 | 35 |
| :---: | :---: | :---: | :---: | :---: |
| Short span (ft) |  | 10 | 15 | 20 |
| $\begin{aligned} & \stackrel{\#}{4} \\ & \frac{\pi}{O} \\ & \stackrel{+}{\pi} \end{aligned}$ | Thickness <br> (in) | 7 | 10.5 | 14 |
|  | Steel (kg) | 1484.456 | 4275.12 | 11315.46 |
|  | Concrete (cft) | 102.08 | 344.53 | 816.67 |
|  | Steel (BDT in thousand) | 74.22 | 213.76 | 565.77 |
|  | Concrete <br> (BDT in thousand) | 25.52 | 86.13 | 204.17 |
|  | Form-work <br> (BDT in thousand) | 4.53 | 15.3 | 36.26 |
|  | Labor (BDT in thousand) | 0.175 | 0.394 | 0.7 |
|  | Total cost (BDT/sft) | 596.86 | 801.47 | 1152.71 |
| $\begin{aligned} & \frac{0}{0} \\ & \frac{\pi}{n} \\ & \frac{0}{0} \\ & .0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | Thickness (in) | 4.5 | 7 | 9 |
|  | Steel (kg) | 2685.44 | 3268.54 | 6118.48 |
|  | Concrete (cft) | 211.85 | 549.49 | 1077.04 |
|  | Steel (BDT in thousand) | 134.27 | 163.43 | 305.93 |
|  | Concrete (BDT in thousand) | 52.96 | 137.37 | 269.26 |
|  | Form-work <br> (BDT in thousand) | 9.406 | 24.397 | 47.82 |
|  | Labor (BDT in thousand) | 0.175 | 0.394 | 0.7 |
|  | Total cost (BDT/sft) | 1124.65 | 826.895 | 891.005 |

Table 4.5: Comparison table for long to short span ratio, $\beta=2.00$

| Long span (ft) |  | 20 | 30 |
| :---: | :---: | :---: | :---: |
| Short span (ft) |  | 10 | 15 |
| $\begin{aligned} & \cong \\ & \stackrel{\pi}{O} \\ & \frac{\pi}{O} \\ & \frac{\pi}{4} \end{aligned}$ | Thickness (in) | 8 | 12 |
|  | Steel (kg) | 1691.51 | 5656.72 |
|  | Concrete (cft) | 133.33 | 450 |
|  | Steel (BDT in thousand) | 84.58 | 282.84 |
|  | Concrete (BDT in thousand) | 33.333 | 112.5 |
|  | Form-work (BDT in thousand) | 5.92 | 19.98 |
|  | Labor (BDT in thousand) | 0.2 | 0.45 |
|  | Total cost (BDT/sft) | 620.144 | 923.924 |
| 0$\frac{0}{0}$$\frac{0}{5}$0000000 | Thickness (in) | 5.5 | 7.5 |
|  | Steel (kg) | 2658.14 | 3801.4 |
|  | Concrete (cft) | 261.707 | 678.81 |
|  | Steel (BDT in thousand) | 132.91 | 190.07 |
|  | Concrete (BDT in thousand) | 65.43 | 169.703 |
|  | Form-work (BDT in thousand) | 11.62 | 30.14 |
|  | Labor (BDT in thousand) | 0.2 | 0.45 |
|  | Total cost (BDT/sft) | 1050.767 | 867.47 |

Table 4.6: Comparison table for long to short span ratio, $\beta=1.24$

| Long span (ft) |  | 31 |
| :---: | :---: | :---: |
| Short span (ft) |  | 25 |
|  | Thickness (in) | 15.5 |
|  | Steel (kg) | 11873.84 |
|  | Concrete (cft) | 10546.88 |
|  | Steel (BDT in thousand) | 594 |
|  | Concrete (BDT in thousand) | 2637 |
|  | Form-work (BDT in thousand) | 469 |
|  | Labor (BDT in thousand) | 7 |
|  | Total cost (BDT/sft) | 532 |
|  | Thickness (in) | 9 |
|  | Steel (kg) | 6980.58 |
|  | Concrete (cft) | 5828.44 |
|  | Steel (BDT in thousand) | 350 |
|  | Concrete (BDT in thousand) | 1458 |
|  | Form-work (BDT in thousand) | 269 |
|  | Labor (BDT in thousand) | 7 |
|  | Total cost (BDT/sft) | 297.1 |
| Conventional slab(Moment Coefficient) | Thickness (in) | 8.5 |
|  | Steel (kg) | 7556 |
|  | Concrete (cft) | 4940 |
|  | Steel (BDT in thousand) | 378 |
|  | Concrete (BDT in thousand) | 1235 |
|  | Form-work (BDT in thousand) | 220 |
|  | Labor (BDT in thousand) | 7 |
|  | Total cost (BDT/sft) | 264 |

Based on the tables shown above, some column-charts and graph were prepared, which are being shown below.


Figure 4.1: Cost comparison chart for $\beta=1.00$


Figure 4.2: Steel comparison chart for $\beta=1.00$


Figure 4.3: Concrete comparison chart for $\beta=1.00$


Figure 4.4: Cost comparison chart for $\beta=1.25$


Figure 4.5: Steel comparison chart for $\beta=1.25$


Figure 4.6: Concrete comparison chart for $\beta=1.25$


Figure 4.7: Cost comparison chart for $\beta=1.50$


Figure 4.8: Steel comparison chart for $\beta=1.50$


Figure 4.9: Concrete comparison chart for $\beta=1.50$


Figure 4.10: Cost comparison chart for $\beta=1.75$


Figure 4.11: Steel comparison chart for $\beta=1.75$


Figure 4.12: Concrete comparison chart for $\beta=1.75$


Figure 4.13: Cost comparison chart for $\beta=2.00$


Figure 4.14: Steel comparison chart for $\beta=2.00$


Figure 4.15: Concrete comparison chart for $\beta=2.00$

We designed only those floor, which have the panels starting from 10 feet to 35 feet. For this reason, only two short spans covered all the values of $\beta$. We developed two graphs for those two short spans ( 10 feet and 15 feet) that compared between different values of $\beta$ and corresponding costs per square feet. Those two graphs are shown below.


Figure 4.16: Cost vs. $\beta$ graph for short span of 10 ft .


Figure 4.17: Cost vs. $\beta$ graph for short span of 15 ft .


Figure 4.18: Cost comparison chart for 31'x25'panel


Figure 4.19: Steel comparison chart for 31'x25'panel


Figure 4.20: Concrete comparison chart for 31'x25'panel

Chapter 5

## CONCLUSION

### 5.1 Limitations

This research work was done based with specifications from ACI 318-11 and BNBC 2006. Only gravity loads, i.e. dead load and live load, had been considered for design. If others loads, aside those two loads, have to be considered, the slabs should be analyzed and designed separately. The floors that had been considered in here, have equal panel sizes. If panel sizes differ, the recommendations of this research cannot be followed but for similar type of floors, i.e. equal spanning floor in same direction. The flat-plates, been designed, were considered having no edge beams. Some of the design data were considered constant, e.g. material strength, bar size, loads, etc. to compare the results evenly.

To mitigate some difficulties in calculations, some approximations were considered in this thesis. So a few limitations have been induced in the results. Such as -

- Conservative calculations were done to estimate material quantities.
- For conventional beam-supported slabs, straight bars were used instead of cranked bars which is more economical solution than straight bars.
- The depth of the beam was assumed as the minimum depth specified by ACI code: table 9.5(a).
- Beam reinforcement was not designed.
- For beam-supported slabs, $\alpha_{\mathrm{fm}}$ has been considered greater than 2.0.
- For beam-supported slabs, punching shear was not checked, which is not needed because of having shear reinforcement in beams.


### 5.2 Recommendations

1. In figure 4.1, 4.2 and 4.3 , it is observed that long span lying between 10 to 20 feet, the total cost were almost equal for both type of slabs; but when the long spans were over 20 feet, the cost for flat plate was excessively higher than the conventional slab. For steel and concrete quantities, similar behavior is observed. So it is being recommended to choose flat-plate only when the long span is up to around 20 feet, when $\beta=1.00$.
2. In figure 4.4, 4.5 and 4.6 , it is observed that if the long span is over than around 12.5 feet, it is better not to choose flat plate but conventional slab, when $\beta=1.25$.
3. In figure 4.7, 4.8 and 4.9, it is observed that if the long span is over than around 20 feet, it is better not to choose flat plate but conventional slab, when $\beta=1.50$.
4. In figure 4.10, it is observed that if the long span is below 26 feet, flat plate is economical, and if the long span is more than that, conventional slab is economical. In figure 4.11, the steel quantity acts as typically stated above. But in terms of concrete comparison (figure 4.12), flat plate is always concrete saver. So it is being recommended that to choose flat plate when the long span is less than 26 feet, and avoid when the greater span needs to be taken into account, when $\beta=1.75$.
5. In figure 4.13, when the long span is around 30 feet the cost is almost equal. In figure 4.14 and 4.15, the behavior in terms of material quantity shows similar pattern to the value of $\beta$ being 1.75. So it being recommended to choose flat plate at the long span being up to 30 feet, when $\beta=2.00$.
6. In figure 4.16, it can be seen that the total cost is almost similar for both systems when $\beta$ is in between 1 to 1.5 for short span being 10 feet. But when the value of $\beta$ exceeds 1.5, the cost increases drastically, the flat plate being more economical. So it is being suggested to use flat plate when $\beta>1.5$.
7. In figure 4.17 (the short span is 15 feet), when the value of $\beta$ is 1 and 1.5 , the cost is almost identical to each other, except for 1.25 where flat plate is more costly. But when $\beta$ lies between 1.5 and 1.75 , the costs do not vary that much. Then after the $\beta$ exceeding above 1.75 , the cost of flat plate is slightly higher than conventional slab. So it is being suggested to choose conventional slab because its rate of change is milder than flat plate.

### 5.3 Conclusion

In a nutshell, it can be said that flat plate is cost effective for the long span being up to 20 feet approximately, when the slab-panels are exactly or almost square shaped. Otherwise, conventional beam-supported slabs are more economical than flat plate, especially for floors with larger spanning panels.

In spite of all of those limitations mentioned in article 5.1, this research work can give a rough idea about the cost comparison between flat-plate and conventional beam-supported slab; and people, both practicing engineers, architects, and non-technical persons, can decide which floor-system is more economical than the other one for what spans and span-ratio; which is the main purpose of this thesis.

This thesis paper will also pave the way for further investigations for more than 35 feet spanned panels, considering other types of floor-systems, e.g. flat-slabs with or without drop-panels/column-capital, one-way ribbed slab, waffle slab, hollow-core slab, advanced composite type slabs, etc.

In this research work, some excel spreadsheets were prepared as mentioned earlier. Those spreadsheets would help the designers and those who are interested in designing flat-plate and beam-supported slabs, to conduct preliminary design in the shortest possible time, with minimum amount of effort.

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